EFFECT OF DIELECTRIC BARRIERS TO THE ELECTRIC FIELD OF ROD-PLANE AIR GAP

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Abstract

The rod-plane gap is extensively used for studies on the breakdown characteristics of gases. In this study, influence of a dielectric barrier on the electric field and potential distributions in a vertically arranged rod-plane gap was numerically analysed by using finite element method (FEM). Maximum field in the gap was examined for different gap distances, size and material of the barrier and positions of the barrier between electrodes for estimating the discharge phenomena. The effect of changing the permittivity of the surrounding dielectric material on the field stress is also studied. The results of the field computation show how the maximum electric field varies with the factors mentioned above. As a result, the highest electric field intensity occurs when the barriers were positioned at the nearest point to the rod electrode and the small sized barriers become effective only in very small air gaps.

1 Introduction

Knowledge of electric fields is necessary in numerous high voltage applications [1]. Electric field analysis provides important roles for the development of design and analysis of high voltage apparatus, as well as the analysis of various discharge phenomena. A typical problem in these applications is to find the electric field between two electrodes. Electrode geometries that are encountered in practice or under laboratory experimental conditions are so large that it is not possible to take into account all. Rod and plane electrodes are a few examples of such electrode geometries.

In some electrode geometries, the electric fields can simply be expressed analytically in a closed form solution; in other, the electric field problem is complex because of the sophisticated boundary conditions, including media with different permittivities and conductivities [2]. For assessing the electric field distributions in the complex arrangements, sometimes in three dimensions, analytical methods are by no means suitable. In such cases the other approaches are in use, numerical techniques and experimental analogs. Numerical methods are particularly useful when the analytical solution is very complicated or impossible.

Electric field problems in high voltage technique are mostly electrostatic field problem. The calculation of electrostatic fields requires the solution of Poisson’s and Laplace’s equations with boundary condition satisfied. Several numerical techniques have been used in the literature for solving Laplace’s and Poisson’s equations for the fields between complex electrode arrangement [3-6]. Finite element method (FEM) is one of the most successful numerical methods for solving electrostatic field problems. FEM can be employed successfully for the computation of an electric field between electrodes in a medium where one or more dielectrics are involved. Solution of the electric field problems by FEM is based on the fact, known from variational calculus, that Laplace’s equation is satisfied when the total energy functional is minimal [7, 8].

In this study influence of dielectric barriers on electric field distribution of a rod – plane gap is examined. Barrier effects have been investigated by a number of authors and many papers have been published on this effect. Most of the papers on this effect include generally experimental studies [9-20] but there is less paper on field analysis of barrier problem [21-24].

In this study finite element method is used to calculate the electric field and potential distributions in the vertically arranged rod-plane gap inserted dielectric barrier. Knowledge of the maximum field is necessary for estimating the discharge phenomena on the rod or in the gap. Maximum fields are examined for different barrier sizes, positions of the barrier and materials of barrier and gap distances. The effect of changing the permittivity of the surrounding dielectric material on the field stress is also studied.
2 Problem Definition and Formulation

In this study, the electric field distribution in a rod-plane gap with a dielectric barrier is numerically analysed by using finite element method (FEM). Figure 1 shows the considered rod-plane electrode configuration with its dimensions. Here, the rod is of 2 mm diameter with a hemispherical end, while the plane disc is of 75 mm diameter with a thickness of 10 mm. The gap is at the vertical position. The geometry of the electrode system has an axial symmetry. Therefore, problem can solve in cylindrical coordinates. In this coordinates, it can be assumed the z-axis of coordinates coincides with the axis of symmetry.

\[ \phi_{10} \]

\[ \phi_{25} \]

\[ \phi_{45} \]

\[ \phi_{75} \]

\[ \phi_{10} \]

\[ \phi_{25} \]

\[ \phi_{45} \]

\[ \phi_{75} \]

Figure 1: Electrode configuration (dimensions in mm)

For applications of ac having extra low frequency as 50/60 Hz or dc voltages, problems may be considered an electrostatic field problem and therefore the electric and magnetic field components may be considered independently of each other, and calculations made on the basis of static field concepts. In the case of electrostatic (and also quasi-static) fields since other electromagnetic fields Maxwell’s equations are the governing equations [7]. In electrostatics, Maxwell’s equations and constitutive equation reduce to the following form

\[ \nabla \times \mathbf{E} = 0 \]  
\[ \nabla \cdot \mathbf{D} = \rho \]  
\[ \mathbf{D} = \varepsilon \mathbf{E} \]  

\[ \nabla \cdot \mathbf{D} = \rho \]  
\[ \nabla \times \mathbf{B} = 0 \]  
\[ \mathbf{B} = \mu \mathbf{H} \]  

where \( \mathbf{E} \) is the electric field intensity, \( \mathbf{D} \) is the electric displacement, \( \rho \) is the space charge density, \( \varepsilon \) is the dielectric permittivity of the material. Based on equation (1), electric field intensity is introduced by the negative gradient of the electric scalar potential \( V \) in following form

\[ \mathbf{E} = -\nabla V \]  

Substituting equations (2) and (3) in (1) Poisson’s scalar equation is obtained as

\[ -\nabla \cdot (\varepsilon \nabla V) = -\nabla \cdot (\varepsilon_0 \varepsilon_r \nabla V) = \rho \]  

where \( \varepsilon_0 \) is the permittivity of free space, \( \varepsilon_r = \varepsilon_r(E, x, y, z) \) is the relative permittivity and \( \rho \) is the space charge density. If the permittivity \( \varepsilon \) is constant such as in the isotropic dielectrics, equation (5) becomes

\[ \Delta V = -\frac{\rho}{\varepsilon} \]  

\[ \Delta V = 0 \]

For space charge free (\( \rho = 0 \)) fields, field is expressed by Laplace’s equation as

\[ \frac{\partial^2 V(r, \theta, z)}{\partial r^2} + \frac{1}{r} \frac{\partial V(r, \theta, z)}{\partial r} + \frac{1}{r^2} \frac{\partial^2 V(r, \theta, z)}{\partial \theta^2} + \frac{\partial^2 V(r, \theta, z)}{\partial z^2} = 0 \]  

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In the case of axial symmetry, the potential distribution is independent of coordinate \( \theta \). Thus, in the cylindrical coordinates, two-dimensional expression of Laplace’s equation is

\[
\frac{\partial^2 V(r,z)}{\partial r^2} + \frac{1}{r} \frac{\partial V(r,z)}{\partial r} + \frac{\partial^2 V(r,z)}{\partial z^2} = 0
\]  

(9)

Solution of equation (9) depends on boundary conditions of the problem. When the problem is solved by FEM, all boundary conditions of the problem must be known. The rod-plane gap is an open boundary problem. It means the field may extend to infinity. Due to symmetry, half of the solution domain of the problem can be analysed. To simplify the calculation, an artificial boundary, shown by the contours in Figure 2, is assumed as the boundary at infinity. According to this, boundary conditions are taken as

\[
V = V_0 = 1 \text{ Volt on the rod},
\]

\[
V = 0 \text{ Volt (ground) on the plane},
\]

\[
\frac{\partial V}{\partial n} = 0 \text{ on all other outer boundaries and on the symmetry axis and}
\]

\[
n \cdot (D_1 - D_2) = 0 \text{ on the surfaces of the dielectric barrier as continuity condition.}
\]

where the normal vector \( n \), points from medium (2) into medium (1)

In the following section, it is explained results obtained by solution of the problem using FEM under conditions given in this section.

3 Results and Discussion

Field of the Rod-Plane Gap without a Dielectric Barrier

Firstly, electric field of the rod-plane gap (Figure 1) without a dielectric barrier is obtained by FEM by applying the initial and boundary conditions. Initial condition applies when there is no charge in the gap before the application of the voltage. We assumed that the applied voltage is 1 Volt across 50 mm gap distance in air at atmospheric pressure. Figure 2 shows the drawn geometry, the discretized mesh of the problem and the computed electric field and potential distribution in the gap without a dielectric barrier from left to right respectively. In this study, numerical computations are performed by using COMSOL Multiphysics 3.3 software based on FEM.

![Figure 2: Problem definition in FEM software and the obtained electric field and potential distributions in the gap without a dielectric barrier](image)

The most important factor influencing the occurrence and characteristics of electrical discharges on a gap is the electric field distribution in the gap. An analysis of discharge performance requires information on the variation of electric field around an electrode and especially maximum electric field on the electrode. If maximum field intensity is greater than electric strength of dielectric medium of the gap, an electrical discharge such as partial discharge, corona or breakdown begins from maximum field point in the dielectric. Therefore, accurate determination of electric field distribution and maximum field is an important prerequisite to the evaluation of the discharge performance of HV apparatus and components. From this point of view, we will interest with maximum field values of electric field.

The results obtained from FEM analysis show that the equipotential lines are condensed around the rod and field distribution in the gap is non-uniform. The field at the rod surface reaches its
maximum value at the point facing the grounded plane. Maximum electric field (Emax) calculations are carried out for different gap distances between 5 mm and 50 mm changed by 5 mm steps. At the end of each computation maximum field value was easily read on surface plot created by the FEM program for electric field distribution and was recorded for drawing of graphics.

Figure 3 shows variation of maximum electric field with gap spacing obtained in the rod-plane gap without a barrier. Emax values are related to 1 Volt (unit potential) of the applied voltage. In the Figure 3 and other figures after this figure, for determination of Emax values at the different voltages from 1 Volt, the Emax values should be multiplied by the applied voltage value in Volt, i.e. if applied voltage is 1000 Volts, 300 V/m in the figure means 300000 V/m. Consequently, maximum electric field on the rod end quickly decreases with the increase in the gap distance as shown in Fig. 3. Small gap distances are critical point of view obtaining higher maximum field.

![Figure 3](image)

**Figure 3: Maximum electric field as a function gap spacing in the rod-plane gap without a barrier**

**Effect of Barrier**

Secondly, a dielectric barrier with a thickness of 3 mm and a diameter of 75 mm is inserted between the rod and the plane electrodes and field distributions for different cases of barrier are examined. In the calculations the dielectric barrier is assumed that is made of polyvinyl chloride (PVC) having relative permittivity of 2.9. Barrier sheet where is parallel to plane was vertically moved between the electrodes for constant gap distance of 50 mm by 5 mm steps. In the Figure 4, it is shown an example for the geometry and mesh of the problem and the obtained electric field distribution and equipotential lines in the gap with the dielectric barrier.

![Figure 4](image)

**Figure 4: Problem definition and the obtained electric field distribution and equipotential lines in the gap with the dielectric barrier**

Figure 4 shows that presence of the barrier in the gap changes the electric field distribution of the gap according to case of field distribution without the barrier. In Figures 5 and 6, variations in the field distribution and equipotential lines with presence of the dielectric barrier are shown. In Figure 5,
the barrier is between the electrodes, in Figure 6, the dielectric barrier is on the plane electrode. Dielectric barrier distorts the field in its neighborhood. Routes of equipotential lines changes by the dielectric barrier due to permittivities of barrier and air are different.

Influence of the barrier on field changes according to type of the applied voltage. For ac voltage applications, free charges are absent at the interface between dielectrics and the polarizations charges define the boundary conditions. On the other hand, in dc voltage applications, accumulation of free charges at the interface takes place due to the differing conductivities of the materials.

In order to demonstrate the effect of presence of the barrier on the electric field, a comparison is performed between the rod-plane gap with and without the dielectric barrier with thickness of 3 mm and diameter of 75 mm (Fig. 7). In this computation the barrier was on the plane. The barrier inserted to the gap has been increased maximum electric field on the rod.
Electrode spacing, s (mm)

Figure 7: Variation of maximum electric field strength with electrode spacing. 1) without the barrier (blue-lower curve); 2) with the barrier (red-upper curve).

Effect of Barrier Size

Figure 8 shows variation of the maximum electric field with thickness of the barrier on the plane as a function of barrier diameter. In the calculations, thickness of the barrier is varied from 3 mm to 50 mm and diameters of the barrier are taken as 37.5 mm, 75 mm and 150 mm respectively.

Figure 8: Variation of maximum electric field strength with thickness of barrier, t for three diameters of the barrier d.
1) d = 37.5 mm (blue-lower curve); 2) d = 75 mm (black curve); 3) d = 150 mm (red-upper curve).

As shown in Figure 8, Emax increases with increase in the thickness of the barrier and increment of the Emax for the barrier having small diameter is higher than bigger one.

Effect of Barrier Distance

Here, distance between the dielectric barrier and the rod was changed between 5 mm and 45 mm by 5 mm steps and maximum field intensities were determined by FEM. If distance between the dielectric barrier and the rod is changed, maximum electric field at the rod end varies. This variation is shown in Figure 9. Variation of the Emax is the highest for the barrier having diameter of 37.5 mm. Generally, Emax decreases with increase in the distance between barrier and the rod. The decrease in Emax for barrier diameter of 37.5 mm is more than ones for barrier of other diameter due to large barriers hinder the plane electrode from the rod. For barrier diameters of 75 mm and 150 mm, variations of Emax are close.
Figure 9: Variation of maximum electric field strength with distance between barrier and rod electrode for three different barrier diameters (d) in the rod-plane gap  
1) d = 37.5 mm (blue curve); 2) d = 75 mm (black curve); 3) d = 150 mm (red curve).

Medium Surrounding Dielectric Barrier

In this case, medium of the gap was changed. In the calculations mentioned above, medium of the gap has been assumed air. The relative permittivity of air is approximately 1. Here, calculations are repeated for a different medium such as transformer oil of $\varepsilon_r = 2.5$. Figure 10 shows results of the calculated Emax. This figure illustrates that Emax on the rod decreases with decrease in the difference between dielectric permittivity of the barrier and dielectric permittivity of the surrounding medium.

Material of Dielectric Barrier

Lastly, to observe the material effect on the electrical stress, five different types of insulating barriers was used which are made of polytetrafluor ethylene (PTFE) ($\varepsilon_r = 2$), polyvinyl chloride (PVC) ($\varepsilon_r = 2.9$), Nylon ($\varepsilon_r = 4$), porcelain ($\varepsilon_r = 5$) and mica ($\varepsilon_r = 6$). Variation of the computed Emax values is shown in Figure 11.
Figure 11: Variation of maximum electric field strength with distance between barrier and rod electrode for the five dielectric barriers having different relative permittivity in the rod-plane gap 
1) $\varepsilon_r = 2$ (PTFE (polytetrafluor ethylene), blue curve); 2) $\varepsilon_r = 2.9$ (PVC (polyvinyl chloride), black curve), 3) $\varepsilon_r = 4$ (Nylon, red curve), 4) $\varepsilon_r = 5$ (porcelain, green curve), 5) $\varepsilon_r = 6$ (mica, pink curve).

Maximum electric field was seen generally on the tip of the rod but sometimes on the plane. Maximum electric field on the plane was in triple point region among plane electrode, barrier and air. Figure 12 shows points of the maximum electric field appeared.

Figure 12: Points of the maximum electric field

During the computational studies, variation of surface charge density on the dielectric barrier with distance from the rod electrode was observed. In the Figure 13, variations of electric charge on the dielectric barrier are shown at distances of 5 mm, 25 mm and 40 mm.

Figure 13: Variation of surface charge density on the dielectric barrier at different distances

4 Conclusion

In this study, variations of the electric field distribution of a rod-plane electrode system with dielectric barrier are analysed at different barrier positions, dielectric materials of barrier, gap spacing. If there is a dielectric barrier in the gap, maximum field intensity increases. Maximum electric field is on the tip of rod and increases for small distances between the barrier and rod.
The maximum electric field was observed when the barriers were positioned at the nearest point to the rod electrode. Also, it is observed that the small sized barriers become effective in very small air gaps.

FEM is highly valued to calculated electric field configurations with fairly simple programs and small computing time. In this study, the field computations were carried out by using COMSOL Multiphysics 3.3 based on finite element method. This useful computer program has the ability to solve the electrostatics and magnetics problems very quickly with high precision.

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References


