Application of Comsol Multiphysics 3.2 to finite strain viscoelasticity of an elastomeric solid.

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Abstract

The paper presents an implementation in Comsol Multiphysics 3.2 of Holzapfel's material model for the viscoelastic stress response of carbon-black filled rubber at large strains. The simulation in Comsol 3.2 uses the Structural Mechanics Module finite strain formulation based on the material configuration with the right Cauchy-Green tensor as a strain measure. The time independent response of the rubber is modelled by the Mooney-Rivlin hyperelastic material with uncoupled volumetric/deviatoric free energy function. In addition to the volumetric and isochoric elastic response function we use a configuration free energy, which drives the viscoelastic response. Thus we obtain a decoupled stress response which consists of equilibrium and non-equilibrium parts. The non-equilibrium viscoelastic stresses are evolving internal variables governed by rate equations which are modelled in PDE General Form. In our simulation we used some predefined SME variables and we defined several other suitable variables such as the isochoric elastic stresses and material tangent modulus. The paper presents three simulations of the elastomeric viscoelastic solids response in relaxation, creep and cyclic loading.

Keywords

Rubber behaviour, Viscoelasticity , Large strains, Stress relaxation, Cyclic loading

1. Introduction

Rubber materials are applied in various branches of mechanical engineering because of their damping properties. The modelling and FEM calculation of the structural response requires a constitutive model which captures the complex material behaviour. The present paper focuses on the viscoelastic behaviour of the filled rubber.

The ground-stress response of filled rubber is usually modelled in the phenomenological framework of finite elasticity by Mooney-Rivlin or Ogden models, or by Aruda and Boyce model in terms of the micromechanically based kinetic theory of polymer chain deformations.

Beside the elastic response the filled rubber shows also the finite viscoelastic overstress response which is apparent in creep and relaxation tests. Cyclic loading tests show a typical frequency-dependent hysteresis as well where the width of the hysteresis increases with increasing stretch rates. The constitutive theory of finite linear viscoelasticity is a major foundation for modeling rate-dependent material behaviour based on the phenomenological approach. In this approach, a suitable hyperelasticity model is employed to reproduce the elastic responses represented by the springs, while the dashpot represents the inelastic or the so-called internal strain. Its temporal behavior is determined by an evolution equation.

2. Model for finite viscoelasticity

The material model of finite strain viscoelasticity used in our work follows from the concept of Simo [2] and Govindjee & Simo [4]. The finite element formulation of the model was elaborated by

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Holzapfel [3] and used by Holzapfel & Gasser [5] to calculate the viscoelastic deformation of fiber reinforced composite material undergoing finite strains.

The model is based on the theory of compressible hyperelasticity with the decoupled representation of the Helmholtz free energy function with the internal variables (Holzapfel [1], p. 283) :

$$\Psi(\boldsymbol{C},\boldsymbol{\Gamma}_{1},\ldots\ldots,\boldsymbol{\Gamma}_{m})=\Psi^{\infty}_{VOL}(J)+\Psi^{\infty}_{ISO}(\overline{\boldsymbol{C}})+\sum_{\alpha=1}^{m}\Upsilon_{\alpha}(\overline{\boldsymbol{C}},\boldsymbol{\Gamma}_{\alpha}),\quad \overline{\boldsymbol{C}}=J^{-2/3}\boldsymbol{C}. \tag{1}$$

The first two terms in (1) characterize the equilibrium state and describe the volumetric elastic response and the isochoric elastic response as $t\rightarrow\infty$, respectively. The third term is the dissipative potential responsible for the viscoelastic contribution. The derivation of the 2nd Piola-Kirchhoff stress with volumetric and isochoric terms:

$$\boldsymbol{S}=2\frac{\partial\Psi(\boldsymbol{C},\boldsymbol{\Gamma}_{1},\ldots\ldots,\boldsymbol{\Gamma}_{m})}{\partial\boldsymbol{C}}=\boldsymbol{S}^{\infty}_{VOL}+\boldsymbol{S}^{\infty}_{ISO}+\sum_{\alpha=1}^{m}\boldsymbol{Q}_{\alpha} \tag{2}$$

where S^{∞}_{VOL} and S^{∞}_{ISO} is the volumetric and the isochoric stress response respectively and the overstress $\boldsymbol{Q}_{\alpha}$ is stress of 2nd Piola-Kirchhoff type.

$$\boldsymbol{S}^{\infty}_{VOL}=J\frac{d\Psi^{\infty}_{VOL}(J)}{dJ}\boldsymbol{C}^{-1}, \tag{3}$$

$$\boldsymbol{S}^{\infty}_{ISO}=J^{-2/3}Dev[2\frac{\partial\Psi^{\infty}_{ISO}(\overline{\boldsymbol{C}})}{\partial\overline{\boldsymbol{C}}}], \tag{4}$$

$$Dev(.)=(.)-1/3[(.):\boldsymbol{C}]\boldsymbol{C}^{-1}. \tag{5}$$

where $Dev(.)$ is the deviatoric operator in the Lagrangian description. Motivated by the generalized Maxwell rheological model (Fig. 1), the evolution equation for the internal variable $\boldsymbol{Q}_{\alpha}$ takes on the form (6).

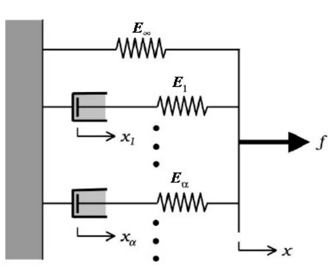

Fig. 1. Maxwell rheological model

$$\dot{\boldsymbol{Q}}_{\alpha}+\frac{\boldsymbol{Q}_{\alpha}}{\tau_{\alpha}}=\dot{\boldsymbol{S}}_{ISO\alpha}, \tag{6}$$

$$\dot{\boldsymbol{S}}_{ISO\alpha}=\beta^{\infty}_{\alpha}\dot{\boldsymbol{S}}^{\infty}_{ISO}(\overline{\boldsymbol{C}}), \tag{7}$$

where

$$\dot{\boldsymbol{S}}^{\infty}_{ISO}(\overline{\boldsymbol{C}})=\frac{\partial\boldsymbol{S}^{\infty}_{ISO}}{\partial\boldsymbol{C}}\dot{\boldsymbol{C}}=\mathbb{C}_{ISO}\dot{\boldsymbol{E}} \tag{8}$$

$$\mathbb{C}_{ISO}=2\frac{\partial\boldsymbol{S}^{\infty}_{ISO}}{\partial\boldsymbol{C}}=2\frac{\partial\boldsymbol{S}^{\infty}_{ISO}}{\partial\overline{\boldsymbol{C}}}\frac{\partial\overline{\boldsymbol{C}}}{\partial\boldsymbol{C}} \tag{9}$$

$\beta^{\infty}_{\alpha}\in(0,\infty)$ in the expression (7) is the nondimensional strain energy factor [2,4], τ_{α} is the relaxation time, $\mathbb{C}_{ISO}$ is isochoric contribution of the tangent elasticity tensor and $\dot{\boldsymbol{E}}$ is the material strain rate tensor

$$\dot{\boldsymbol{E}}=\frac{1}{2}(\dot{\boldsymbol{F}}^{\boldsymbol{T}}\boldsymbol{F}+\boldsymbol{F}^{\boldsymbol{T}}\dot{\boldsymbol{F}}). \tag{10}$$

The material is assumed slightly compressible, the volumetric and isochoric (Mooney - Rivlin) parts of Helmholtz free energy function were chosen in the form

$$\Psi^{\infty}_{VOL}(J)=\frac{1}{d}(J-1)^2, \quad \Psi^{\infty}_{ISO}(\overline{\boldsymbol{C}})=c_1(\overline{I}_1-3)+c_2(\overline{I}_2-3), \tag{11}$$

with the parameters $c_{1,}c_2$ and *d*. The viscoelastic behavior is modeled by use of $\alpha=2$ relaxation processes with the corresponding relaxation times τ_α and free energy factors β^{∞}_{α}. All the parameters were determined from experimental measurements.

3. Finite element simulation in Comsol Multiphysics

The material model described above was implemented into Comsol Multiphysics. The Structural Mechanics and PDE modules were used for the calculation of time dependent response of a rubber block in plain strain in different loading regimes. The implementation is very similar to the Viscoelastic material case in Structural Mechanics Model Library. The application mode type plane strain in Structural Mechanics Module, the time dependent analysis and the Mooney-Rivlin hyperelastic material were chosen. The components of the isochoric stress rate $\dot{\boldsymbol{S}}^{\infty}_{\boldsymbol{ISO}}$ were determined in Symbolic Toolbox in Matlab and added to the scalar expression table in Comsol. PDE module was used for the integration of the evolution equation (6). The results of the different simulations show the good qualitative agreement with experimental time dependent behaviour of filled rubbers. The similar model based on Prony series was implemented into ANSYS 10 [6] and the results of simulations are comparable with our ones.

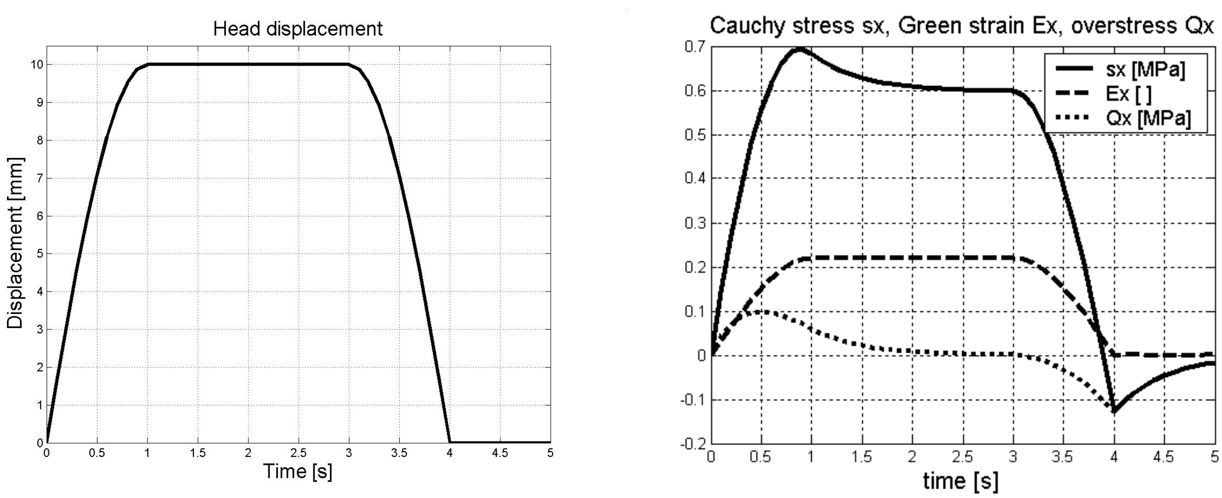

Fig. 2. Time dependent displacement controled loading of rubber block

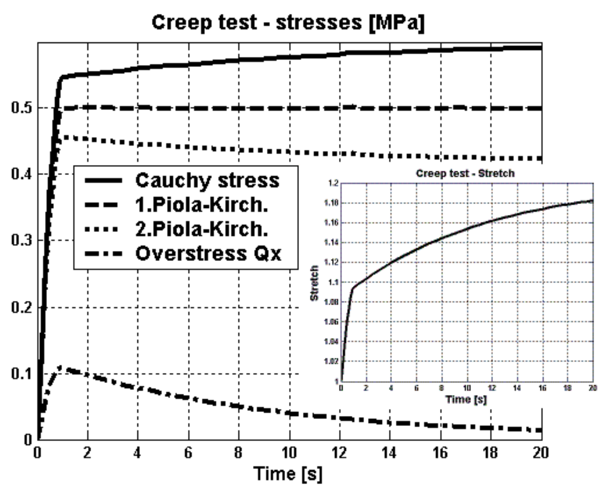

Fig. 3. Creep test

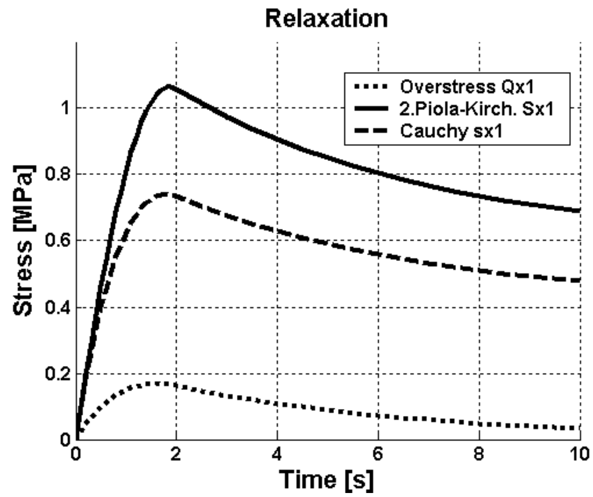

Fig. 4. Relaxation test

4. Conclusion

The paper has presented the FEM implementation in Comsol Multiphysics of a viscoelastic material model in finite strain in the Lagrangian configuration. Simple examples were subsequently presented, namely the relaxation, creep and time dependent loading of a rubber block. The block was modelled using Structural Mechanics and PDE modules.

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