

# Space-Phase Modelling of Electromechanical Processes in Motion Machines. 2D Model Realization

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## Abstract

**At modeling of electromechanical conversion the analysis of an electromagnetic field in inhomogeneous- motion to a medium is fulfilled, calculation of force interactions between skew fields and integrated parameters of this field. The basis of a phase class of methods of model operation of electromechanical processes new spatially designed. Outcomes of computing experiments about spatially phase models of the elementary two-phase electrical machine without a collector with plane-parallel magnetic and an electric field are submitted.**

It is known that electromechanical energy conversion occurs when real bodies are in motion in electromagnetic field (EMF). The important stages in modeling of such conversion are analysis of electromagnetic field in non homogenous motion medium, accounts of force interactions between bodies as well as accounts of this field integral parameters. There are two large classes of electromagnetic field analysis – space temporary (ST) and space frequency (SF). The first of these classes covers a variety of dynamic processes in electromagnetic fields but requires too great expenditures of computing resources and that is why often is of little use for generation of control algorithms by electromechanical devices operating in electric drives, mechanotronic modules, electrogenerating complexes etc.. The second class of methods strictly speaking is suitable only for analysis of electromagnetic fields in motionless linear medium. The known means of these methods spreading on nonlinear and non homogenous moving mediums are not strict enough and are very limited in application [1].

The authors elaborated the basis of a new space-phase class of electromechanical processes modeling methods. The essence of these methods is in process modeling at the fixed state of analyzing system that allows to exclude time from equations. SP methods are based on transformation ST of the equations of mathematical physics in SP shape. It gives a possibility to carry out the analysis of transients only at an integrated level, not calculating transients in EMF.

It is possible to write down an initial ST equations of EMF in differential form according to mathematical definition of a full derivative of vectorial and scalar field on time in an inhomogeneous field of velocities:

$$\frac{d\mathbf{F}}{dt} = \frac{\partial\mathbf{F}}{\partial t} + \text{rot}(\mathbf{F}_H \times \mathbf{v}) + \mathbf{v} \text{div} \mathbf{F}_H = \frac{\partial\mathbf{F}}{\partial t} + (\mathbf{v}\nabla)\mathbf{F}_H + \mathbf{F}_H \text{div} \mathbf{v} - (\mathbf{F}_H \nabla)\mathbf{v}, \quad (1)$$

$$\frac{d\Phi}{dt} = \frac{\partial\Phi}{\partial t} + (\mathbf{v}\nabla)\Phi_H, \quad (2)$$

where  $\mathbf{F}$  – the analyzable vector field in inhomogeneous motion medium;  $\mathbf{F}_H$  – the analyzable vector field in homogeneously motion (fixed) medium;  $\mathbf{v}$  – the field of velocities describing mechanical motion of all bodies in an analyzable system;  $\Phi$  – the analyzable scalar field in inhomogeneous motion medium;  $\Phi_H$  – the analyzable scalar field in homogeneously motion (fixed) medium. Formula (1) represents "data-flux" definition of a full derivative on time, i.e. such definition at which the equation of a law of electromagnetic induction differential form for motion mediums in an exactitude coincides with the relevant equation for the fixed mediums at replacement of a partial derivative on time for the full. The formula (2) represents the formal application of an operator (1) to a scalar field. According to definitions (1) and (2) all vectors EMF (except for which are concerned to be homogeneously motion medium), included in equations of mathematical physics should be represented in frame of references, is rigid the bound with points of observation which are moving according to assigned fields of velocities and partial accelerations.

The problem of mathematical physics of analysis EMF in an inhomogeneous field of velocities may be formulated by transformation of Maxwell equations with the material equation according to definitions (1) and (2).

$$\operatorname{rot} \mathbf{H} = \boldsymbol{\delta} + \gamma \mathbf{E}, \quad \operatorname{rot} \mathbf{E} = -\frac{d\mathbf{B}}{dt}, \quad \mathbf{H} = \nu_a (\mathbf{B} - \mathbf{B}_r) + q_1 \frac{d}{dt} (\mathbf{B} - \mathbf{B}_r) - q_2 \frac{d^2}{dt^2} (\mathbf{B} - \mathbf{B}_r), \quad (3)$$

where  $q_1$  и  $q_2$  – dynamic parameters of electro physical properties of a periodical structure materials magnetoconductors. The linearized material equation (3) is noted concerning effective vectorial values of a magnetic intensity and a magnetic induction. The set of equations (3) according to (1) is needed to be supplemented a similar system for vectors EMF in the fixed system of bodies.

Dynamics of a variation of all physical quantities in SP methods is taken into account by entering of scalar and vector fields of phase variables. Such principle of the analysis allows essentially to reduce need for computing resources of the computer, does not superimpose any restrictions on a linearity or nonlinearity of physical properties of substance, and also on character of mechanical motion of all bodies which are participating in electromechanical transformation of an energy. All physical quantities are defined in inhomogeneous motion frame of reference, is rigid the bound with a field of velocities, private accelerations etc. The aspect of the obtained equations of mathematical physics does not contradict a principle of a relativity of motion of Galileo. Hence, the given class of methods is invariant in relation to a choice of a global inertial system of a reference of mechanical motion of bodies.

In a new class of methods calculation of transients in EMF is not required, and dynamic properties of electromechanical transformers are simulated in space of states for electrodynamic potentials. Input datas of models are: geometric exposition of electromagnetic system, parameters of material properties of an electromagnetic system details, distribution of variable states of EMF sources. Output datas are points of a set of dynamic characteristics of the electromechanical transformer represented by functional dependents of the electromagnetic moment, operating on a rotor, and voltages on all windings from an angular position of a rotor and currents in all windings, and also their derivatives on time up to the order  $n$  inclusive. For the majority of practically significant cases usually there is enough a first or second order.

Below we shall present results of computing experiments with spatially phase models of the elementary two-phase the electrical machine without collector with a plane-parallel magnetic field (currents are guided perpendicularly calculated plane) and a plane-parallel electric field (by virtue of a symmetry of a system the requirement of a short-circuit on planes is satisfied, parallel calculated and indefinitely removed from it). For a minimizing of a stated material volume in models we shall consider dynamic effects only the first order.

Modeling is carried out in program complex COMSOL Multiphysics. This complex in essence differs from other software intended for a solution of problems of mathematical physics, that typical forms of the partial differential equations (PDE) have the greatest generality, and there are unlimited possibilities of their combination at build-up of set of equations. In COMSOL Multiphysics it is supported three forms PDE: Coefficient, General and Weak. The General form is represented to the most convenient for modeling since it does not demand finishing up to the logic end of a deduction of PDEs and enables to feature nonlinearities of physical properties of substance easily enough. Here briefly we shall present results of modeling.

Let's enter a system of phase variable electrodynamic potentials at the fixed state of an electro-mechanical system [2]. Zero phase variable are potentials. The first phase variables have dimensionality of derivatives on time from zero variables, but are not equal this derivative.

The system bidimensional SP the equations which are taking into account dynamic effects first order, looks like

$$\begin{aligned} \operatorname{rot} \mathbf{H}_H &= \boldsymbol{\delta} + \gamma \mathbf{E}_H, & \operatorname{rot} \mathbf{H}_{H1} &= \boldsymbol{\delta}_1, & \operatorname{rot} \mathbf{H} &= \boldsymbol{\delta} + \gamma \mathbf{E}, \\ \operatorname{rot} (\mathbf{H}_1 + 2(\mathbf{H}_H \nabla) \mathbf{v} + \mathbf{H}_H \times \operatorname{rot} \mathbf{v} - \mathbf{H}_H \cdot \operatorname{div} \mathbf{v}) &= \boldsymbol{\delta}_1 \end{aligned}$$

$$\mathbf{H}_H = v_a (\mathbf{B}_H - \mathbf{B}_r) + q_1 \frac{d}{dt} (\mathbf{B}_H - \mathbf{B}_r), \quad \mathbf{H} = v_a (\mathbf{B} - \mathbf{B}_r) + q_1 \frac{d}{dt} (\mathbf{B} - \mathbf{B}_r), \quad (4)$$

$$\mathbf{H}_{H1} = v_a (\mathbf{B}_{H1} - \mathbf{B}_r), \quad \mathbf{H}_1 = v_a (\mathbf{B}_1 - \mathbf{B}_r).$$

The set of equations (4) is supplemented with relations, defining phase variables in the fixed and a motion frame of reference of bodies [2], and after reduction to the scalar form is entered in COMSOL Multiphysics. The system (4) is solved concerning phase variables of a vectorial magnetic potential.

Figure 1 is showing the schematical image of a cross-section of the two-phase machine. Numerals designate numbers of subdomains of computing area: 1 – the positive cut of the first phase of a winding stator; 2 – the positive cut of the second phase of a winding stator; 3 – the negative cut of the second phase of a winding stator; 4 – the negative cut of the first phase of a winding stator; 5 – outside air; 6 – magnetoconductor of stator; 7 – an air clearance between stator and a rotor; 8 – a rotor. Sectoring of coordinate axes is given in millimeters.

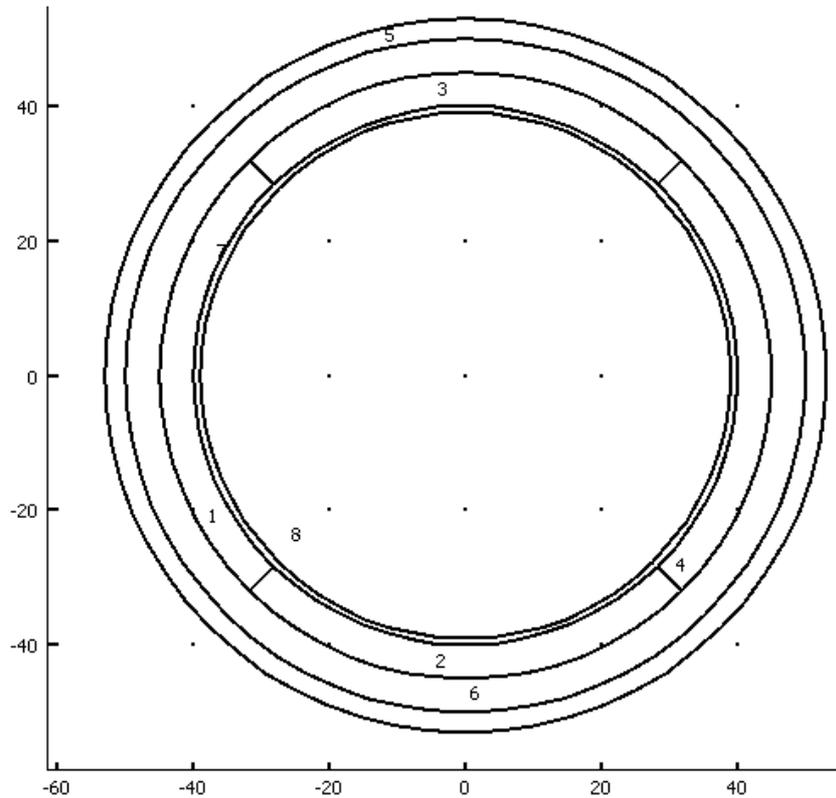


Figure 1: The schematical image of a cross-section of the machine

We shall consider the asynchronous machine which rotor represents copper cylindrical solid body with a specific electrical conductance  $\gamma = 0.056 \text{ MSm/mm}$ . Let the machine work in a condition of a dynamic brake if through the first phase of a winding stator the direct current which density  $\delta = 1 \text{ A/mm}^2$  (the magnetic moment upwards). Solving a set of equations (4), we shall calculate velocity performances of the electromagnetic moment operating on a curl, and electromotive force, directed in windings stator. We see, that the electric field is directed only in the field of a rotor and in a transitional band (in a clearance), and in the fixed part of the machine the electric field is not directed, hence, there is no electromotive force. Figure 2 is showing the calculated velocity performance of the electromagnetic moment. The positive direction of gyration and the electromagnetic moment – counter-clockwise. It is visible, that the brake moment is proportional to a rotation frequency of a rotor. On figure 3 the pattern of allocation of a vectorial magnetic potential, and on figure 4 - allocation of the first phase variable of this potential is shown at a rotation frequency of a rotor  $n = 10 \text{ Turn/s}$ . It is visible, that the magnetic field is deformed rotaried copper solid body, and the electric field focused only in a rotaried part of the machine – in a rotor and in a transitional band.

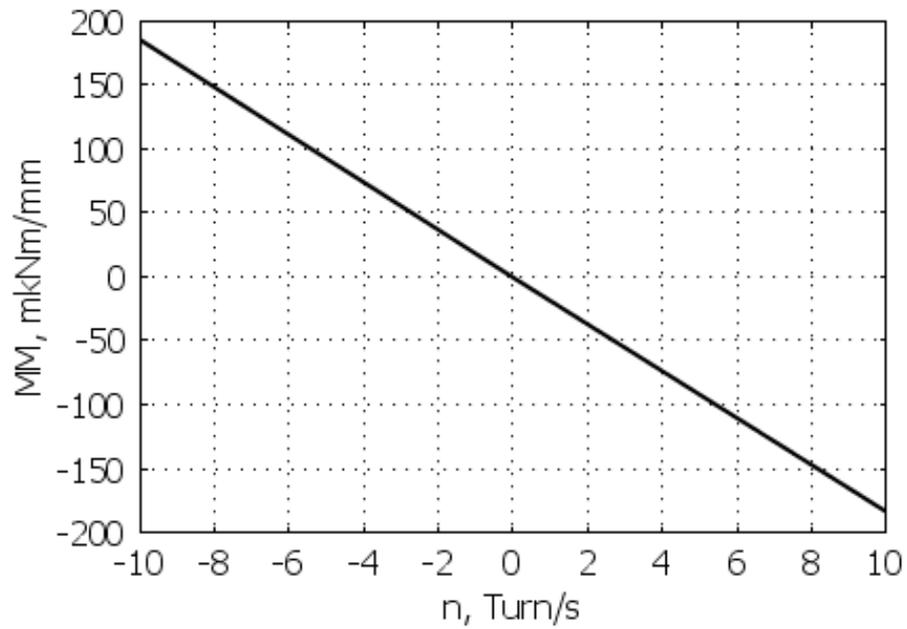


Figure 2: Velocity performance of the electromagnetic moment

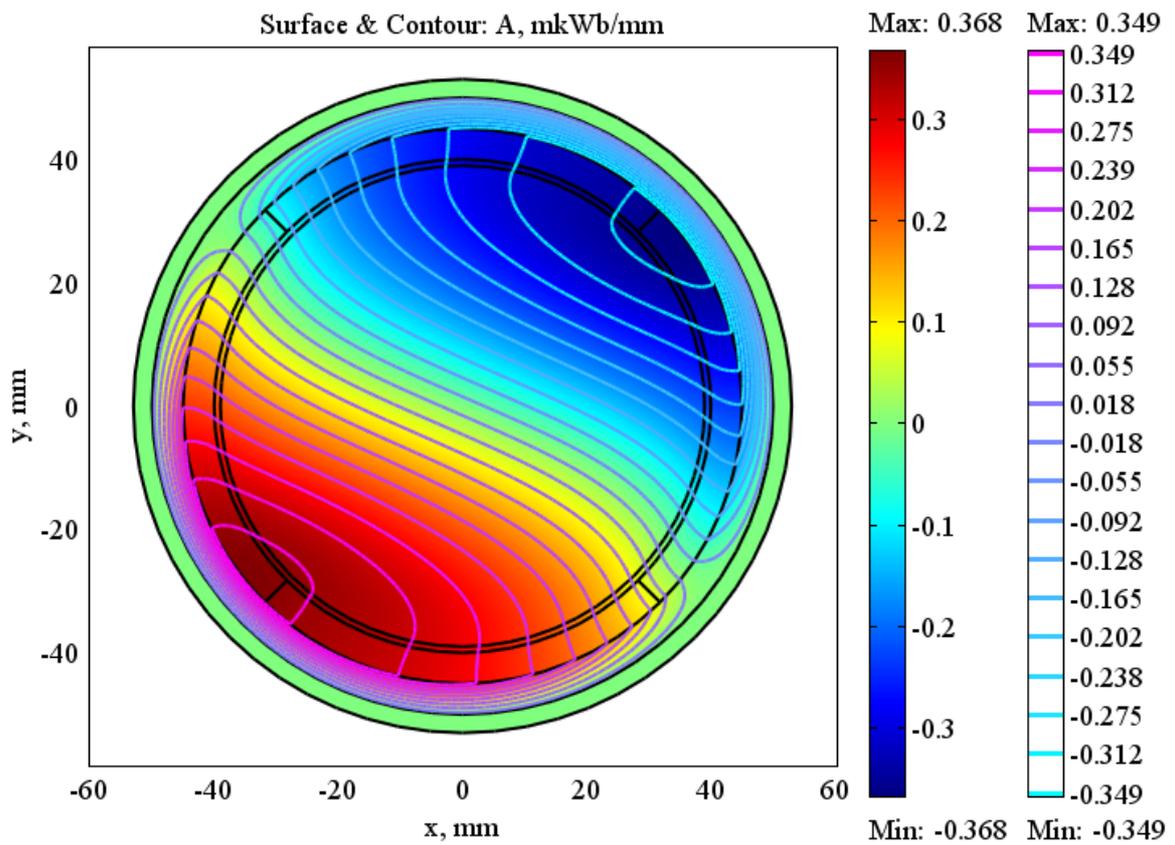


Figure 3: Distribution of a vectorial magnetic potential

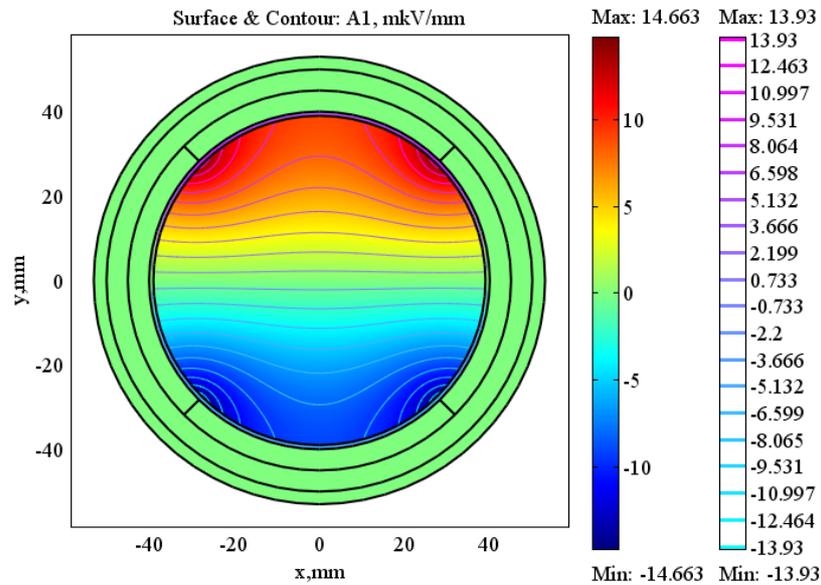


Figure 4: Pattern of isolines of the first phase variable of a vectorial magnetic potential

Now we shall consider the synchronous machine which rotor is a permanent magnet with a residual magnetic induction 1 Tl, directional along an axis  $x$ , and the specific electrical conductance is equal 0.01 MSm/mm. We shall set equal to zero all phase variables of phase stator currents. Let the magnet rotates counter-clockwise with frequency  $n = 10$  Turn/s. Distribution of a vectorial magnetic potential in the given system does not represent interest since it can be calculated in usual magnetostatic model. Distribution of the first phase variable of a vectorial magnetic potential is computing. It is equal to an electric field strength with opposite character. This distribution is shown on figure 5. It is visible, that the electric field exists only in the fixed part of the machine and in a transitional band. The electric field induces an electromotive force in phase windings stator. Figure 6 is showing the angular performance of an electromotive force on one coil in these windings (the first phase – a solid curve, second – dashed).

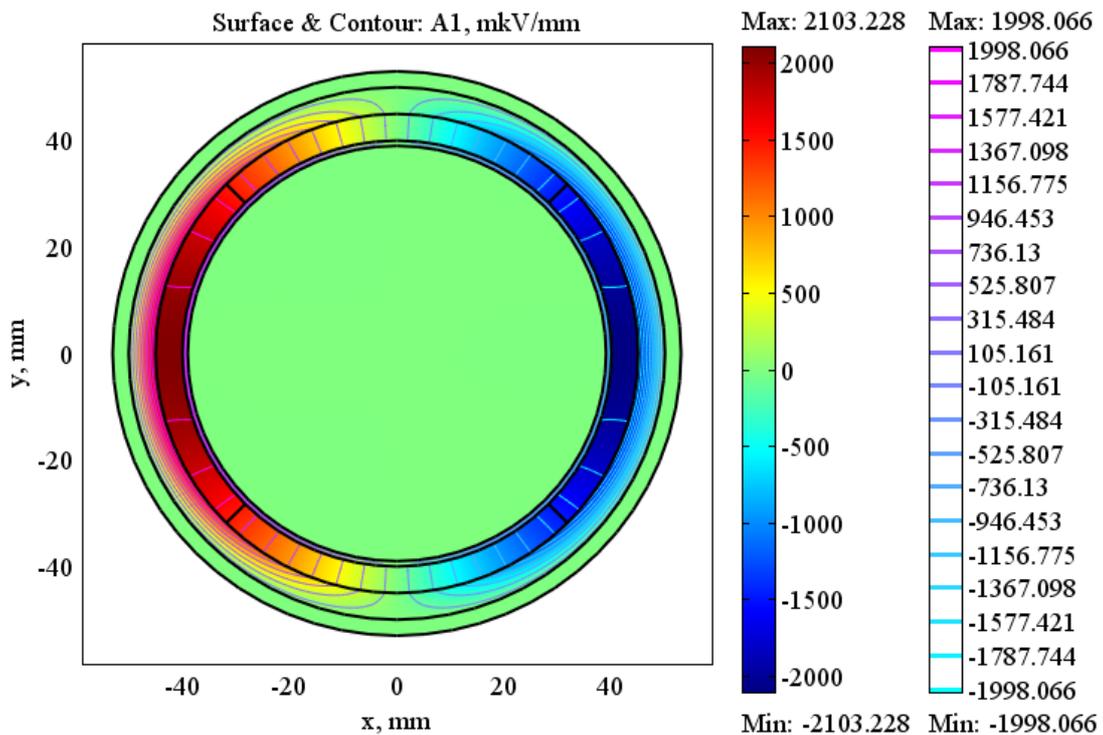


Figure 5: Distribution of the first phase variable of a vectorial magnetic potential

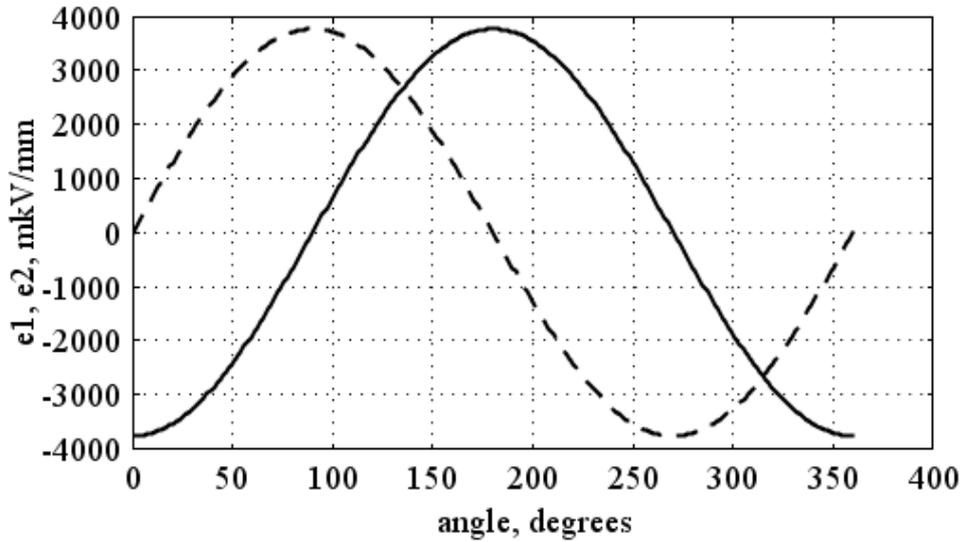


Figure 6: Angular performance of electromotive force in stator windings

Again we shall return to already surveyed asynchronous machine. Let the rotor rotates with frequency 10 Turn/s. In terms of dynamic effects of the first order we shall simulate gyration of a magnetic moment of stator with various frequencies  $f$ : from 0 up to 20 Hz. Let the direction of gyration of a magnetic moment of stator coincides with a direction of gyration of a rotor. We shall calculate velocity performance of the electromagnetic moment operating on a rotor at a modification of frequency of a stator current. Let in a considered instant the current density in the first phase is equal  $1 \text{ A/mm}^2$ , and the velocity of its modification is equal to zero. The current density is equal the second phase to zero, and the velocity of its modification is equal  $-2 \cdot \pi \cdot f$ . Dependence of the electromagnetic moment on a rotation frequency of a moment of magnet (i.e. from frequency of a stator current) is shown on figure 7. It is visible, that on synchronous frequency the electromagnetic moment is a little bit less than zero (on 4.75 % from a value of the moment at the fixed rotor). It means, that for exposition of dynamic effects of electromechanical transformation it is not enough two phase variable electrodynamic potentials.

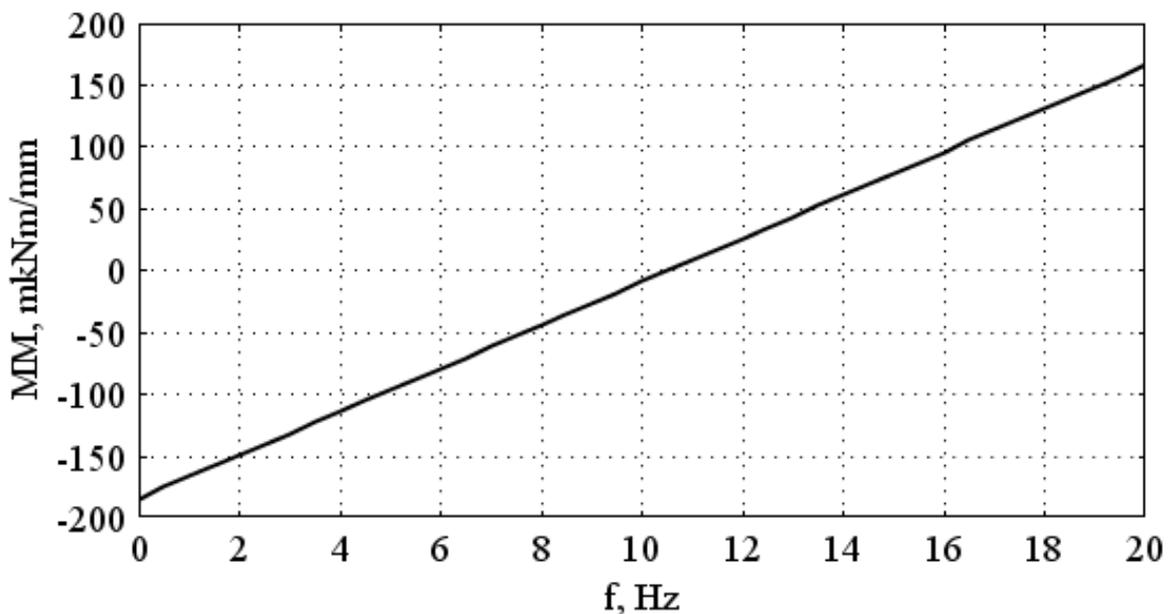


Figure 7: Velocity performance of the electromagnetic moment

Results of model operation do not contradict the known facts in electromechanics. All performances of dynamic properties of transformers in space of states represent functional connections of an aspect [2]:

$$\{M_M, [U]\} = f\left(\alpha, \frac{d\alpha}{dt}, \frac{d^2\alpha}{dt^2}, [I], \frac{d[I]}{dt}, \frac{d^2[I]}{dt^2}\right). \quad (5)$$

All reference performances of electrical machines of a rotation are particular cases of performances of an aspect (5), for example mechanical performance, angular performance of the moment, performance of a no-load operation, loading and adjusting performances of drives and generators. Performances of an aspect (5) are directly bound to energy indexes of operation of the machine. On performances (5) it is simple to build algorithms of control by the machines, providing greatest possible values of energy indexes (efficiency, an electrical power factor etc.). This implies that SP methods of model operation of processes of electromechanical transformation of energy are rather useful to complex projection of electric drives and generating complexes as a whole, including a dynamoelectric and electronic parts.

## References

- [1] Kopylov I.P. Mathematical modeling of electrical machines. – M.: The higher school, 2001. – 588 p.
- [2] Shmelev V.E., Sbitnev S.A. Spatially- phase model operation of electromechanical processes in machines with stationary values magnets in a system finite element calculations FEMLAB. – Electrical engineering, 2004, # 8. – p. 50–54.

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