The Czech Treasury Yield Curve from 1999 to the Present

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Abstract

I introduce Czech Treasury yield curve estimates at a daily frequency from 1999 to the present. I use a well-known and simple yield curve model that is shown to fit the data very well. The estimation of the model parameters is based on market prices of Czech government bonds. The estimated parameters can be used to calculate spot rates and hence par rates, forward rates or discount function for practically any maturity. The spot rates are fundamental financial variables. However, to my knowledge consistent time series of the spot rates are not available for the Czech economy. I believe that my work will fill this gap and the presented yield curve estimates will be used by both practitioners and researchers. My resulting data will be posted on Internet and periodically updated.

1 Introduction

The yield curve is a fundamental determinant of almost all asset prices. The yield curve also influences many economic decisions. The Czech Treasury is by far the biggest issuer of bonds denominated in Czech koruna and thus the Treasury yield curve is a natural benchmark yield curve of the Czech economy. However, consistent yield curve estimates over a long time period are not available. I estimate, day by day, the Czech Treasury yield curve from beginning of 1999 to the present. The yield curve can be expressed in terms of spot rates, par rates, forward rates or discount function. I carefully check the estimated curves and, in details, evaluate the estimation error.

I use parametric models of (Nelson and Siegel, 1987) and (Svensson, 1995) to infer the Treasury yield curve from government bond prices. The Nelson-Siegel model, which has only 4 parameters, enables us to estimate the yield curve, without being overparameterized, when the number of observed bond prices is very small. The Svensson model, which adds two more parameters to the Nelson-Siegel model, is capable to fit more complicated shapes of the yield curve, particularly both the "hump" at short maturities rates and the concave shape at long maturities. Despite the parsimonious number of parameters, the presented models prove to fit the data very well and thus provide accurate picture of the Czech Treasury curve.

This paper is motivated by (Gurkaynak, Sack and Wright, 2006), who use Svensson model to estimate the U.S. Treasury Curve from 1961 to the present. (Slavík, 2001) is the first to use the Nelson-Siegel model to estimate the Czech Treasury yield curve from government bond prices. Another example is (Málek, Radová and Štěrba, 2006), who uses Svensson model to comment on predictive ability of model implied forward rates. Both authors estimate and assess the yield curve for only one, respectively two particular dates. In contrast, I run the estimation

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routine for more then 2 600 days and evaluate the estimation results in both cross-section and time series perspective.

This paper is organized as follows. Section 2 reviews fundamental concepts of interest rates and reminds important relations in fixed income analysis. Section 3 presents the Nelson-Siegel modeling framework, including estimation issues. Section 4 overviews our data. Section 5 demonstrates the yield curve estimation for two arbitrarily chosen dates and defines estimation error measures. Section 6 present estimation results over the ten year period, including an assessment of the quality of the fit. Section 7 concludes. The resulting data are posted as an data appendix to this paper.

2 Yield Curve Basics

Let me first review fundamental concepts and relations of interest rates that will be used in subsequent discussion. More details can be found, for example, in (Cairns, 2004).

2.1 The Discount Function, Zero-coupon Bonds and Spot Rates

The key element in asset pricing is the discount function, or the price of a zero-coupon bond. Let $\delta_t(\tau)$, the discount function, denote the price at time t of a zero-coupon bond that pays 1 Czech koruna at the maturity date $t+\tau$. We use τ to denote the time to maturity, we speak about τ-period zero bond.

The continuously compounded spot interest rate, or spot yield or zero-coupon yield for a zero-coupon bond maturing in τ periods ahead is related to the zero-coupon bond price by

$$r_t(\tau) = -\frac{\ln(\delta_t(\tau))}{\tau} \tag{1}$$

and conversely the zero-coupon price, or the discount function, can be written in terms of spot rate as

$$\delta_t(\tau) = e^{-r_t(\tau)\tau}. \tag{2}$$

Although the continuously compounded interest rates may be mathematically convenient, interest rates are often expressed on a coupon-equivalent basis, in which case the compounding is assumed to be annual instead of continuous. The discount function is then expressed as

$$\delta_t(\tau) = \frac{1}{(1+r_t^{ce}(\tau))^\tau}, \tag{3}$$

where $r_t^{ce}(\tau)$ is the coupon-equivalent, or annually compounded spot interest rate. One can easily derive the relation between the continuously compounded yield and the annually compounded yield:

$$r_t(\tau) = \ln(1+r_t^{ce}(\tau)). \tag{4}$$

The yield curve, or the term structure of interest rates at a given date t is unambiguously represented by a set of the spot interest rates with different maturities.

2.2 Coupon Bonds, Yield to Maturity, Par Rates and Bootsrapping

Bonds are almost solely issued as coupon-bonds in practice. Given the discount function, we can price any coupon bond by summing the price of its individual cash flows. For example, the price $P_t(\tau_n)$ at time t of a τ_n-period coupon-bearing bond that pays a face value 100 Czech koruna at the maturity date $t+\tau_n$ and has n coupon payments left, where each coupon payment has a nominal value C Czech koruna ($C = 100c$, where c is a coupon rate) and the last coupon payment is at the maturity date is as follows:

$$P_t(\tau_n) = \sum_{i=1}^{n} C\delta_t(\tau_i) + 100\delta_t(\tau_n), \tag{5}$$

where $\delta_t(\tau_i),\ i = 1, \ldots, n$ are discount functions (zero-coupon bond prices) with maturities $\tau_1, \ldots, \tau_n$.

Bond prices can be quoted in two different forms. The bond price $P_t(\tau_n)$ in (5) is called a dirty price. The dirty price is the actual amount paid when buying the bond. The clean price is an artificial price which is, however, the most often quoted price in markets. It is equal the dirty price minus the accrued interest. The accrued interest is equal to the amount of next coupon payment multiplied by the proportion of elapsed period from previous coupon, respectively from issue date in case of the bond's first coupon payment. The clean price is used because it does not jump at the time a bond goes ex-coupon. See (Cairns, 2004), pp. 3–4 for details and Figure 1.2 on p. 4 for a graphical illustration.

For coupon bonds, yields to maturity are often quoted on markets. The yield to maturity is the constant interest rate that discounts the bond's cash flows so that they equal to the price of the bond. Hence the annualy compounded yield to maturity y_t for the coupon bond from equation (5) fulfills

$$P_t(\tau_n) = \sum_{i=1}^{n} \frac{C}{(1+y_t)^{\tau_i}} + \frac{100}{(1+y_t)^{\tau_n}}, \tag{6}$$

Yields to maturity are often used to represent τ-period interest rate. But the picture they provide is imprecise. First, the yield to maturity is a measure of a bond's implied average interest rate, if it is held to maturity. This measure implicitly assumes that all coupon payments are reinvested at this same average interest rate. Second, for a given yield curve (term structure of spot rates), the yield to maturity for a bond with a given maturity will depend on its coupon rate. Therefore two coupon bonds that mature at the same date have different yields to maturity if they have different coupon rates. For these reasons yields to maturity should not be used to represent the yield curve. Instead spot rates or par rates should be used.

A par interest rate, or par yield is the coupon rate $c_t(\tau_n)$ at which a τ_n-period coupon bond would trade at par. Hence according to the pricing equation (5) it must satisfy

$$100 = 100c_t(\tau_n) \sum_{i=1}^{n} \delta_t(\tau_i) + 100\delta_t(\tau_n). \tag{7}$$

This implies that

$$c_t(\tau_n) = \frac{1 - \delta_t(\tau_n)}{\sum_{i=1}^{n} \delta_t(\tau_i)}. \tag{8}$$

An example of par rates, quoted on markets, are interest rate swaps. A swap is an agreement between two counterparties to exchange cash flows in the future. In interest rate swap, one counterparty pays cash flow equal to interest at a predetermined fixed swap rate on a notional principal for a number of years. In return, it receives interest at a floating rate on the same notional principal for the same period of time.

Let us observe a set of swap rate quotes for maturities $\tau_1 = 1, \tau_2 = 2, \ldots, \tau_n = n$ years. This par rate quotes assume annual coupons. For example, $c_t(10)$ denotes par rate quote of 10-year interest rate swap that matures exactly in 10 years at date $t + 10$ and has 10 coupon payments at dates $t + 1, t + 2, \ldots, t + 10$. In this, in practice rather special case, when we have quotes of coupon rates related to all coupon payment dates, we can determine the discount function from par rates by manipulating (8):

$$\delta_t(\tau_{j+1}) = \frac{1 - c_t(\tau_{j+1}) \sum_{i=1}^{j} \delta_t(\tau_j)}{1 + c_t(\tau_{j+1})}, \quad j = 1, \ldots, n-1 \tag{9}$$

and $\delta_t(\tau_1) = 1/(1 + c_t(\tau_1))$. This recursive procedure, which converts par rates to discount functions, is called bootstrapping. We can proceed further from (9) to determine spot rates. In practice, we have to adjust (7) and (9) to take into account day count conventions.

2.3 Duration, Convexity and Convexity Bias

Duration is a central figure in fixed-income analysis. For a coupon-bond it measures time (in years) that the bond holder must wait to receive bond's cash flows. It is a weighted average of times when the cash flows pay out, with weights equal to the cash flows discounted by the yield to maturity:

$$D = \frac{1}{P_t(\tau_n)} \left(\sum_{i=1}^{n} \frac{\tau_i C}{(1+y_t)^{\tau_i}} + \frac{\tau_n 100}{(1+y_t)^{\tau_n}} \right), \tag{10}$$

where $P_t(\tau_n)$ is the price of the τ_n-period coupon-bond, y_t is the annually compounded yield to maturity defined by (6) and τ_i measures time to maturity in years. Equation (10) implies that for a given maturity, the higher the coupon, the shorter the duration. Further note that a zero-coupon bond has duration equal to its time to maturity. The market practitioners often work with the following duration definition

$$D^M = \frac{D}{1+y_t} \tag{11}$$

which is referred to as a modified duration. Modified duration is used as the sensitivity measure of a relative price change to the yield to maturity change.[2] In context of spot rates, the yield to maturity change can be thought as a parallel shift of the spot yield curve. The first-order Taylor expansion of bond price (6) with respect to the y_t results in

$$\frac{\Delta P(\tau_n)}{P(\tau_n)} \approx -D^M \Delta y_t, \tag{12}$$

where $D^M = -(1/P(\tau_n))\mathrm{d}P(\tau_n)/\mathrm{d}y_t$ is the modified duration and $\Delta P(\tau_n) = P(\tau_n, y_t + \Delta y_t) - P(\tau_n, y_t)$. This first-order approximation is, however, accurate only for small changes in yield to maturity because the relation between price and yield to maturity is nonlinear.

Convexity captures this nonlinearity. The second-order Taylor expansion of the bond price with respect to the y_t results in

$$\frac{\Delta P(\tau_n)}{P(\tau_n)} \approx -D^M \Delta y_t + \frac{1}{2} C (\Delta y_t)^2, \tag{13}$$

where $C = (1/P(\tau_n))\mathrm{d}^2 P(\tau_n)/\mathrm{d}y_t^2$ is the convexity of the bond. Convexity implies that the capital loss from an increase in interest rates will be smaller than the capital gain from a decline in interest rates. In particular, long-period bonds exhibit very high convexity[3] which tends to depress long-period interest rates. This impact of convexity is referred to as the convexity bias. The convexity bias can be one of the main reasons for the noticeable concave shape of the yield curve at long maturities. An example of the convexity bias is shown in Section 5.1.

2.4 Forward Rates

Finally, the yield curve can also be unambiguously expressed in terms of forward rates rather than spot rates or par rates. Forward interest rates or forward yields are interest rates between times $t + \tau_1$ and $t + \tau_2$ in future ($\tau_2 > \tau_1$) implied by current spot rates. In other words we are fixing the interest rate between times $t + \tau_1$ and $t + \tau_2$ in advance at time t.

The forward rate $f_t(\tau_1, \tau_2)$ can be synthesized from spot rates (zero-coupon bond prices) by simple no-arbitrage argument. Consider a contract, traded at time t, in which we agreed to lend to an investor 1 Czech koruna at time $t + \tau_1$ for receiving back $e^{f_t(\tau_1,\tau_2)(\tau_2-\tau_1)}$ Czech koruna at time $t+\tau_2$. The forward contract imposes cash flows of -1 at time $t+\tau_1$ and $+e^{f_t(\tau_1,\tau_2)(\tau_2-\tau_1)}$ at time τ_2. To determine $f_t(\tau_1, \tau_2)$, we replicate this contract by borrowing $e^{-r_t(\tau_1)\tau_1}$ Czech koruna

[2] A price sensitivity measure is often the only purpose of duration in the area of interest rate derivatives. The original definition of duration as a weighted average of times when the instrument's cash flows pay out is then meaningless.

[3] Convexity increases roughly as a square of duration.

at time t until time $t+\tau_2$ and by immediately depositing this amount under the spot interest rate $r_t(\tau_1)$ until time $t+\tau_1$. This deposit gains exactly 1 koruna that we lend to the investor at time $t+\tau_1$. Further note that we have to pay back $e^{-r_t(\tau_1)\tau_1}e^{r_t(\tau_2)\tau_2}$ koruna at time $t+\tau_2$. By definition, the forward contract has zero value at time t when the contract is closed. Thus, the $e^{-r_t(\tau_1)\tau_1}e^{r_t(\tau_2)\tau_2}$ amount must be equal to the $e^{f_t(\tau_1,\tau_2)(\tau_2-\tau_1)}$ amount we receive from the investor. By solving this equality we obtain the continuously compounded forward rate:

$$f_t(\tau_1,\tau_2) = \frac{r_t(\tau_2)(\tau_2-\tau_1) - r_t(\tau_1)\tau_1}{\tau_2-\tau_1}. \tag{14}$$

Alternatively, we can express the forward rate in terms of zero-coupon bond prices (discount function):

$$f_t(\tau_1,\tau_2) = \frac{1}{\tau_2-\tau_1}\ln\left(\frac{\delta_t(\tau_1)}{\delta_t(\tau_2)}\right). \tag{15}$$

The instantaneous forward rate is defined as limit

$$f_t(\tau) \equiv \lim_{\tau_2\to\tau_1} f_t(\tau_1,\tau_2). \tag{16}$$

Note, that in this case τ denotes time to settlement of the instantaneous interest rate. The definition of a derivative of a continuous function applied on (15), makes it possible to express the instantaneous forward rates as

$$f_t(\tau) = -\frac{\partial\ln(\delta_t(\tau))}{\partial\tau} = -\frac{1}{\delta_t(\tau)}\frac{\partial\delta_t(\tau)}{\partial\tau}. \tag{17}$$

Thus we can relate zero-coupon bond price to instantaneous forward rates by evaluating the integral of (17) from t to $t+\tau$:

$$\delta_t(\tau) = \exp\left(-\int_{u=t}^{t+\tau} f_t(u)\mathrm{d}u\right). \tag{18}$$

By applying the spot interest rate definition (1) on (18), we can express spot interest rate as an average of instantaneous forward rates:

$$r_t(\tau) = \frac{1}{\tau}\int_{u=t}^{t+\tau} f_t(u)\mathrm{d}u. \tag{19}$$

Further, we can differentiate the definition of the spot rate (1) with respect to τ and use the relation (17) to write:

$$f_t(\tau) = r_t(\tau) + \tau\frac{\partial r_t(\tau)}{\partial\tau}. \tag{20}$$

This relation tells us that forward rates are above spot rates if the yield curve is upward sloping, and vice versa.

2.5 Day Count Conventions

Czech Government Bonds are issued as coupon-bonds with coupon payments once a year. For calculating cash flow time periods the 30E/360 day count convention is used. The 30E/360 convention assumes that a year has 12 months of 30 days each. The time period τ between dates $D_2/M_2/Y_2$ and $D_1/M_1/Y_1$ (read $Day/Months/Year$) is then calculated as

$$\tau = \frac{360(Y_2-Y_1)+30(M_2-M_1)+(D_2-D_1)}{360}. \tag{21}$$

If D_1 is equal to 31, it is changed to 30 before plugging into (21). If D_2 is equal to 31, it is changed to 30 before plugging into (21). Prague interbank deposits market PRIBOR and Czech interest rate swaps use the Act/360 day count convention. Under this convention, the time period τ is calculated as an actual number of days between the two dates divided by 360.

3 The Yield Curve Model

If the Treasury issued a full spectrum of zero-coupon bonds every day, then we could simply observe the yield curve on the market. Unfortunately, this is not the case. In the Czech Republic, only coupon-bonds are issued by the Treasury and the number of bonds issued is very limited – the maximum number of the Czech government bonds traded on the market at the same time is 16. Hence, we need a yield curve model to infer spot rates from prices of existing bonds. Models proposed for estimating spot rates and hence forward and par rates fit some function of time to maturity to observed coupon-bond prices. A spot rate is then a function of time to maturity and model parameters.

3.1 Nelson-Siegel Framework

I will use Nelson-Siegel type of models to estimate forward and thus spot rates from observed coupon-bond prices. To simplify the notation, I drop the time subscript t in forward rates, spot rates and parameters in this section. (Nelson and Siegel, 1987) assumes that the instantaneous forward curve is the solution to a second-order differential equation with two equal roots:

$$f(\tau) = \beta_0 + \beta_1 e^{-\lambda\tau} + \beta_2 \lambda\tau e^{-\lambda\tau}, \tag{22}$$

where $\theta = (\beta_0, \beta_1, \beta_2, \lambda)$ is a vector of parameters. Parameters β_0 and λ are restricted to be positive. The forward rate (22) is a three-component exponential function. The first component, β_0, is a constant to which the forward rate approaches, when the time to settlement approaches to infinity. The second component, $\beta_1 e^{-\lambda\tau}$, is a monotonically decreasing (or increasing, if β_1 is negative) exponential term, and the third component, $\beta_2\lambda\tau e^{-\lambda\tau}$, can generate a "hump".

To increase the flexibility and improve the data fit, (Svensson, 1995) extends the Nelson-Siegel model by adding another "hump" component and thus increasing the number of parameters to six:

$$f(\tau) = \beta_0 + \beta_1 e^{-\lambda_1\tau} + \beta_2 \lambda_1\tau e^{-\lambda_1\tau} + \beta_3 \lambda_2\tau e^{-\lambda_2\tau}, \tag{23}$$

where $\theta = (\beta_0, \beta_1, \beta_2, \beta_3, \lambda_1, \lambda_2)$ is a vector of parameters. Parameters β_0, λ_1 and λ_2 are restricted to be positive.

3.2 Spot Rate Curve and Its Behavior

The spot rates of the Nelson-Siegel modeling framework are derived by integrating the forward rates (22) and (23) according to (18). The corresponding Nelson-Siegel spot rate curve is

$$r(\tau) = \beta_0 + \beta_1 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_2 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right). \tag{24}$$

The corresponding Svensson spot rate curve is

$$r(\tau) = \beta_0 + \beta_1 \left(\frac{1 - e^{-\lambda_1\tau}}{\lambda_1\tau}\right) + \beta_2 \left(\frac{1 - e^{-\lambda_1\tau}}{\lambda_1\tau} - e^{-\lambda_1\tau}\right) + \beta_3 \left(\frac{1 - e^{-\lambda_2\tau}}{\lambda_2\tau} - e^{-\lambda_2\tau}\right). \tag{25}$$

Again, the spot rate (24) is a three-component exponential function. The first component, β_0, is a constant to which the the spot rate approaches, when the time to maturity approaches to infinity. The second component, $\beta_1(1 - e^{-\lambda_1\tau})/(\lambda_1\tau)$, is monotonically decreasing (or increasing, if β_1 is negative) and governs the slope of the yield curve. This component approaches to 0, when time to maturity approaches to plus infinity and it approaches to β_1 when time to maturity approaches to 0. The third component, $\beta_2((1 - e^{-\lambda\tau})/(\lambda\tau) - e^{-\lambda\tau})$, starts at 0, increases (or decreases, if β_2 is negative) and then approaches back to 0, when time to maturity approaches to infinity. Thus, this component can be viewed as the component, that can generate a "hump". The Svensson's fourth component behaves in exactly the same way and allows to fit a second "hump" of the spot rates curve. Components properties pin down the limiting behavior of spot rates:

$$r(0) = \beta_0 + \beta_1 \quad \text{and} \quad r(\infty) = \beta_0 \tag{26}$$

(Diebold and Li, 2006) offers an interpretation of the Nelson-Siegel yield curve from a different point of view. If we set λ to be some constant, we could consider betas (β_0, β_1, β_2) as factors and the terms, by which we multiply betas are then factor loadings. This interpretation is in line with results of Principal Component Analysis (PCA). The PCA applied on the yield curve shows that first three main components (factors) usually explain over 99% variability in interest rates. The first three comopnents are then called level factor, slope factor and curvature ("hump") factor. This result is consistent accros different data sets and time periods. According to this interpretation, the Nelson-Siegel model can be considered as a three factor model and Svensson model is a four factor model. For an example of the PCA applied on the Czech swap curve, see (Kladívko, Cícha and Zimmermann, 2007).

The parameter λ or λ_1, governs the speed of the decay of the second component and hence the steepness of the curve. Small values of λ or λ_1 produce slow decay and can better fit the curve at long maturities, while large values of λ or λ_1 produce fast decay and can better fit the curve at short maturities. λ or λ_1, also determines a possible "hump" of the third component and Svensson's λ_2 determines a "hump" of the fourth component.

My empirical experience with the Svensson model shows that the third component (governed by λ_1) indeed fits a "hump" (usually observable for shorter maturities), but the fourth component (governed by λ_2) is often utilized to fit the convexity bias (concave shape of the curve) which goes in hand with capturing a large proportion of the yield curve slope. This situation typically occurs when estimating long-period yield curves.

I graphically demonstrate the model components for two arbitrarily chosen settlement dates in Figure 1. The corresponding estimated yield curves are presented in Section 5.1. The left chart presents the Nelson-Siegel model components, when the maximum maturity used in estimation is 4.6 years. The components meet their theoretical interpretation. The right chart presents the Svensson model components, when the maximum maturity used in estimation is 27.9 years. In this case, the fourth component fits the convexity bias apparent at the long end of the curve and thus explains a large proportion of the yield curve slope (see the corresponding zero curve plotted in the right chart of Figure 3).

3.3 Estimating the Yield Curve Model

In estimating the forward rate curve and hence the spot rate curve, I choose parameters to minimize the weighted sum of squared deviations between the observed and the model implied prices of coupon-bonds:

$$\widehat{\theta} = \arg\min_{\theta} \sum_{i=1}^{N} \left(\frac{P_i - \widehat{P}_i}{D_i^M} \right)^2 , \tag{27}$$

where N is a number of observed bonds, P_i is the observed dirty price of the coupon-bond, $\widehat{P}_i$ is the model implied price (estimated price) of the coupon-bond and $1/D_i^M$ is an optimization weight. The model implied price $\widehat{P}_i$ is calculated by plugging spot rates given by (24) or by (25) into the discount function (2) and then summing the bond's cash flows according to (5). The time fraction of cash flows is computed according to (21). Thus, I infer continuously compounded spot rates under the 30E/360 day count convention.

I follow (Gurkaynak, Sack and Wright, 2006) and set the optimization weights equal to the inverse of modified duration defined by (11). Using these weights, we utilize the relation (12) and approximately minimize the sum of squared deviations between the observed and the model implied yields to maturity.[4] It is possible to directly minimize sum of squared deviations between

[4] We should set weights equal to $1/(D_i^M P_i)$ to exactly obey the first order approximation. However, when doing so, we get into numerical problems as the weights appear to be to small numbers. Of course, this can be solved by rescaling all the weights by some constant. Or we can proceed the way utilized in this paper, i.e. "forget" the P_i in the weights, since it is roughly the same number ($P_i \approx 100$ for 100 notional principal).

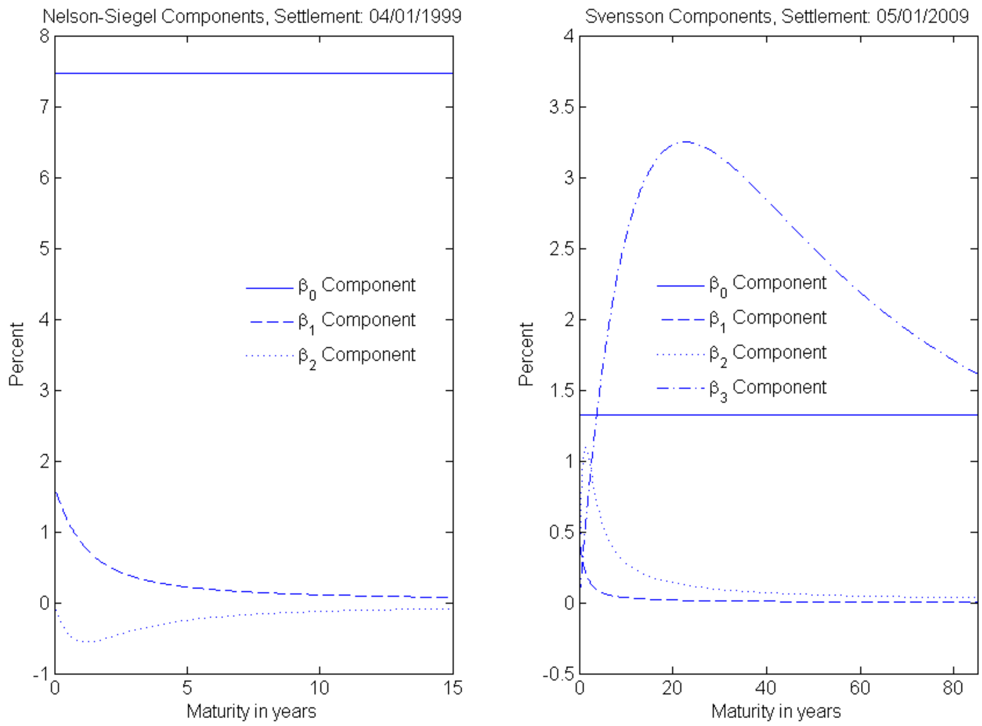


Figure 1: The left chart plots components of the Nelson-Siegel model. The right chart plots components of the Svensson model. The time to maturity is set to values which exceed the maximum maturity of the bonds used for estimation (4.6 years for the Nelson-Siegel model and 27.9 years for the Svensson model). The reason for this is to show the limiting behavior of the components.

the observed and the model implied yields to maturity, which results in virtually the same parameter estimates. But it is computationally inconvenient, since yield to maturity is calculated by numerically solving nonlinear equation, which is time consuming.

The estimation based on fitting yields to maturity implies roughly equal mismatch of predicted versus observed yields to maturity, irrespective of maturity. The other estimation strategy is to minimize unweighted sum of squared deviations between the observed and the model implied prices. This strategy often ends up with large mismatch of predicted versus observed yields to maturity at the short end of the yield curve. This is because of smaller duration of short-period bonds, which causes their yields to maturity to be more sensitive to price changes. The pros and cons of fitting yields to maturity versus fitting prices is discussed, for example, in (Slavík, 2001) or (Svensson, 1995).

It is possible to develop a maximum likelihood estimator to obtain standard errors of parameter estimates. One needs to assume some distribution of bond price errors and than calculate the covariance matrix of parameter estimates. In my opinion, this extra work will not bring much benefit because we often deal with very small number of observations.

3.4 The Estimation Procedure Implementation

The estimation procedure represents a nonlinear least squares problem. I rely on MATLAB's `lsqnonlin` optimization algorithm in my implementation. Unfortunately, I can not guarantee that I succeed in finding the global minimum. The objective function (27), in which the estimated price $\widehat{P_i}$ is given by Nelson-Siegel type of model is known to have multiple local minima, see (Cairns, Pritchard, 2001) for details. Further, the fit of the model can be almost equally good for different combinations of parameters.

The final values of parameter estimates are sensitive to the initial values supplied to the optimization algorithm, especially for the Nelson-Siegel's λ and Svensson's λ_1 and λ_2. I handle this problem in a standard way – I run the estimation on a grid of different initial values of lambdas. I set the grid of lambdas to cover a possible hump on the whole range of maturities used in estimation. In case of the Svensson model, I set the grid of λ_1 to cover the "hump" at short maturities and I set the grid of λ_2 to improve the fit at very long maturities.[5] Note, that to avoid multicollinearity λ_1 and λ_2 must be different. If the optimization results in the same values of λ_1 and λ_2, it suggests that the Svensson model appears to be overparametrized and should be replaced with the Nelson-Siegel model, which is a special case of the Svensson model.

The estimation of betas appear to be less sensitive to initial values compared to "more nonlinear" lambdas. I set the initial values of β_0 and β_1 according to the relation (26), in which I replace $r(0)$ with the shortest period yield observed and $r(\infty)$ with the longest period yield observed.

4 Data and Estimation Issues

4.1 Bond Price Data

I use Czech Government Bonds prices collected by the Prague Stock Exchange (PSE). For every outstanding bond, the PSE averages the end of day price quotes delivered by the Czech government bond market makers on each business day.[6] The PSE started to do so on July 14, 1997. In this paper, I present results based on average of bid and ask prices, i.e. on mid prices.

In my opinion the PSE data set contains very credible quotes, which reflect the real market prices for each business day. I have checked the PSE price quotes with available Reuters and Bloomberg data sets and found no substantial differences. It is important to note, that neither Reuters nor Bloomberg possess complete bond price data over such a long period of time. In the entire PSE data set, I have found 120 price quotes missing. Fortunately, the missing observations are just a one-day quotes skips for a given bond. I replace the missing price quote with its value from the preceding day.

While the PSE data set starts in mid 1997, I run the yield curve estimation from January 4, 1999 (first business day in 1999). The reason is insufficient number of bonds available before 1999. I am forced to exclude all bonds issued before January 1, 1998 because they were issued under different taxation regimes. The taxation regime influences bond price and thus bonds with different taxation regimes can not be put together into one yield curve. At least four bonds should be used to identify the four parameters of the Nelson-Siegel model and four bonds are available shortly before the end of 1998.

Starting January 4, 1999, I include in the estimation all government bonds with the following exceptions:

- I exclude all bonds with less than 180 days to maturity, since their price quotations often can not be considered to represent the real prices. I suspect, this is caused by their low liquidity.
- I exclude all bonds before they reach 30 days after the issue date. This rule concerns only the first tranche for the given bond. Again, in some cases the price quotes of the new bond issues behave oddly for the first few weeks.
- I exclude the 6.08 %/2001 bond (issue number 27), since this bond appears to be constanlty over priced. This becomes obvious by just visually checking the yield to maturity curves – the yield to maturity of this bonds is apparantly too low. When used in estimation, its observed yield lies as much as 60 basis points below the fitted yield and the difference

[5]For example, for $\lambda = 0.1$, the "hump" achieves its maximum at the maturity of 17.9 years, for $\lambda = 10$, the "hump" achieves its maximum at the maturity of 0.22 years.

[6]The rules called Determination of an Average Reference Price for a Bond are published as a PSE electronic document, downloadable from: `ftp://ftp.pse.cz/Info.bas/Eng/Rules/bond_price.pdf`

between the observed and fitted yield stays negative for the whole life of the bond cosidered in estimation.

- I exclude the 4.85 %/2057 bond (issue number 54). This 50-year bond is a low volume issue, which is not actively traded on the market.
- I also exclude floating interest rate bonds, since their use in yield curve estimation is not straightforward.

All the bonds used in estimation are listed in Table 2 in Appendix A. In total, I use 28 943 bond price observations spanning the period of 2 611 business days. In Figure 2, I plot the number of bonds used in estimation and the corresponding maximum time to maturity of the bonds.

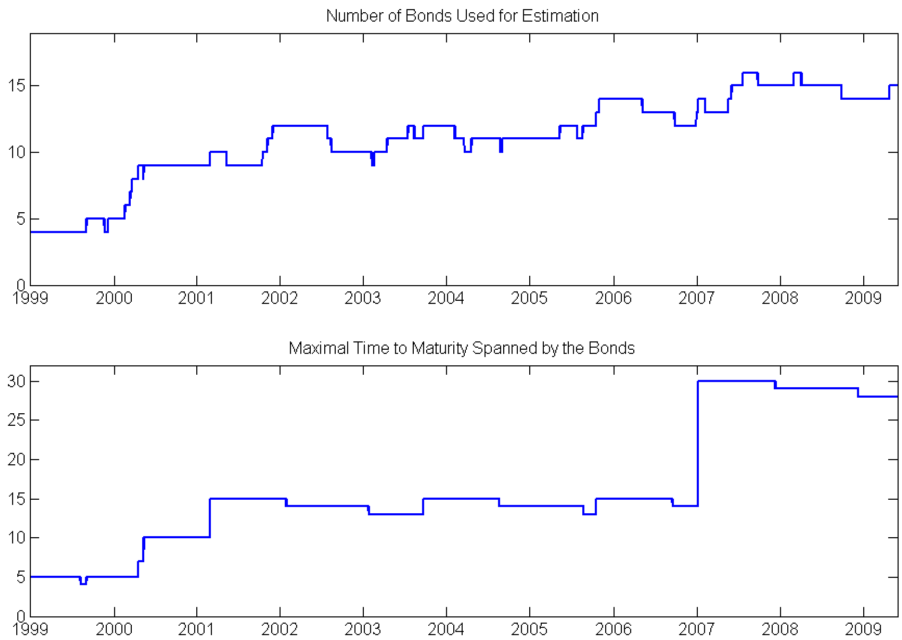


Figure 2: The upper chart displays number of bonds used in estimation. The lower chart displays the maximum maturity of the bonds.

4.2 Short End of the Yield Curve

I find it important to fit the very short end of the Nelson-Siegel and Svensson parametric curve. Otherwise, in some cases the short end can end up with unreasonable values, such as negative rates or extremely high rates. Even though the estimated short-period interest rates are not to be used by the model users, they are employed in the estimation routine to discount coupon cash flows. Thus it is important to keep their values in reasonable range. I fix the short end of the yield curve with the arithmetic average of PRIBOR and PRIBID overnight rates, which I refer to as PRIBOR MID overnight. I transform PRIBOR MID overnight rates into zero-coupon bond prices with just one day to maturity.

One can argue that the interbank deposits market is rather disconnected from the government bonds market. I see two reasons for using PRIBOR MID overnight rate. First, Treasury overnight deposits steadily attract interest that is fixed a few basis points above PRIBID overnight rate.[7] Second, the PRIBOR (PRIBID) overnight rate is probably the only credible

[7] I checked this on several tens of Depo and Repo contracts made by the Debt and Financial Assets Management Department of the Czech Treasury in years 2003-2009. The Czech Treasury has been using Depo and Repo contracts to manage the liquidity of the state budget. I am not permitted to release the Depo and Repo data.

and consistent source of the Czech overnight rate available on a daily basis. The disadvantage of using PRIBOR MID overnight rate is its high volatility which is translated further up into the short end of the yield curve. I revisit this issue in Section 6.4.

5 Estimation Example and Error Measures

5.1 Yield Curve Fitting on January 4, 1999 and January 5, 2009

In Figure 3 I present spot, instantaneous forward and par rates estimated for two arbitrarily chosen dates, January 4, 1999 (first day of my data set) and January 5, 2009 (first business day of current year). The inferred rates are transformed, using the relation (4), to the annual

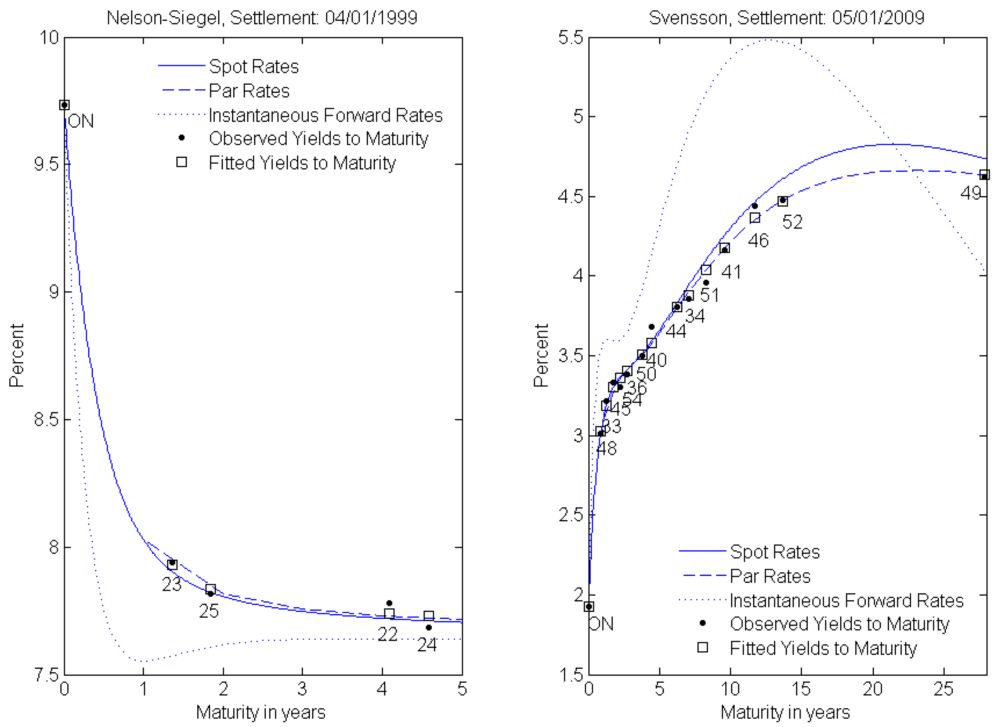


Figure 3: Estimated spot, par and forward rates (annually compounded). Nelson-Siegel model was used for January 4, 2009, Svensson model for January 5, 2009. Estimated according to the objective function (27). The number displayed under the yield to maturity mark is the issue number of the bond. The issue number is a unique identifier which is incremented by one for every new issuance of the Czech government bond. See Table 2 in Appendix A for a list of bonds used in estimation. The mark ON denotes the PRIBOR MID overnight rate.

compounding, which is usually preferred by market practitioners. I use the Nelson-Siegel model for January 4, 1999 and the Svensson model for January 5, 2009. There are only 4 government bonds available for inferring the rates on January 4, 1999, with the maximum maturity of 4.6 years. Situation is much better 10 years later – there are 15 government bonds available on January 5, 2009 with the maximum maturity of 27.9 years.

We can visually check the fit by comparing observed (dots) and fitted (boxes) yields to maturity. The difference between the observed yield to maturity and the fitted (model implied) yield to maturity is the estimation error. In other words, the error is the residual between the observed and fitted value. We can see from Figure 3 that the yield curve models do excellent job fitting the entire cross-section of yields to maturity of government coupon-bonds for both dates. The success at fitting the yield to maturity of government bonds is repeated throughout our entire data set.

The yield curve is inverted on January 4, 1999, which is rather untypical shape for the Czech Treasury curve. The yield curve is steeply rising ten years later. The Svensson model is able to fit the hump at maturities from 2 to 6 years and the convexity bias at the long end. The fit is worst for the 3.70%/2013 bond (issue number 40) which apperas to be underpriced relative to the neighboring bonds. This "scattered" bond is a relatively frequent event during the current financial market crisis. I revisit this issue in Section 6.1.

5.2 Error Measures

The estimation error can be also expressed as a difference between observed and fitted (model implied) prices. The estimation error may indicate that the bond is mispriced. Further, the error represents idiosyncratic noise, which is not captured by the model. The noise could arise from liquidity issues, non-synchronous quotes or data errors for different bonds.

To summarize the estimation error, I report the Root Mean Squared Error (RMSE) and Maximum Absolute Error (MaxAE). The RMSE and MaxAE for the yield to maturity error are calculated as follows:

$$\begin{aligned} \text{RMSE} &= \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \widehat{y}_i)^2}, \\ \text{MaxAE} &= \max_i\{|y_i - \widehat{y}_i|\}, \quad i = 1, \dots, n, \end{aligned}$$

where n is the number of observations (number of bonds plus number of short-period rates) for a given settlement date, y_i is the observed yield to maturity and $\widehat{y}_i$ is the fitted yield to maturity. Price errors are calculated by replacing the yield to maturity values with price values.

The error measures for January 4, 1999 and January 5, 2009, are reported in Table 1, which confirms the very plausible fit apparent from Figure 3. The Maximum Absolute Error for the yield to maturity is just 5.1 and 10.5 basis points, respectively.

Table 1: This table reports Root Mean Squared Error (RMSE) and Maximum Absolute Error (MaxAE) for both price and yield to maturity errors. The number in parenthesis next to the MaxAE value is the corresponding issue number of the bond. Yield to maturity errors are measured in basis points. Price errors are measured in Czech haléře for 100 Czech koruna notional principal.

	Yield to Maturity		Price	
Settlement	RMSE	MaxAE	RMSE	MaxAE
4 January 1999	2.8	5.1 (24)	11	18 (24)
5 January 2009	4.4	10.4 (40)	26	63 (46)

6 Estimating the Czech Treasury Yield Curve from 1999 to the Present

I run, day by day, the cross-section estimation of the yield curve described in Section 3.3 from beginning of 1999 until present. I use the four-parameters Nelson-Siegel model to estimate the yield curve from January 4, 1999 till December 29, 2000 and since then, I use the six-parameters Svensson model. I have at least two reasons for using the more parsimonious Nelson-Siegel model for the first two years of our data set. First, until the beginning of 2000 the Nelson-Siegel model is the only choice because the number of available bonds stays bellow 6, see Figure 2, and thus parameters of the Svensson model could not be identified.[8] Second, the Svensson model

[8] I use the PRIBOR MID overnight rate in the yield curve estimation, so only 5 bonds are actually needed to econometrically identify the Svensson model. Five bonds are available from December 6, 1999. However, recall

overfits the yield curve in several days of 2000. The two "hump" components appear too close to each other which creates unrealistic "humps" between neighboring bonds. Since beginning of 2001 the problem with overfiting of the Svensson model disappears and also the Svensson model starts to clearly outperform the Nelson-Siegel model in terms of the error measures. On February 26, 2001 the 15-year 6.95%/16 bond (issue number 34) is used in estimation and the second "hump" component of the Svensson model fits the concave shape of the yield curve, as the convexity bias becomes more important.

6.1 Analysis of Error Measures Evolution

I illustrate the day by day evolution of Root Mean Squared Error (RMSE) and Maximum Absolute Error (MaxAE) for yields to maturity in Figure 4. The longitudinal statistics of the

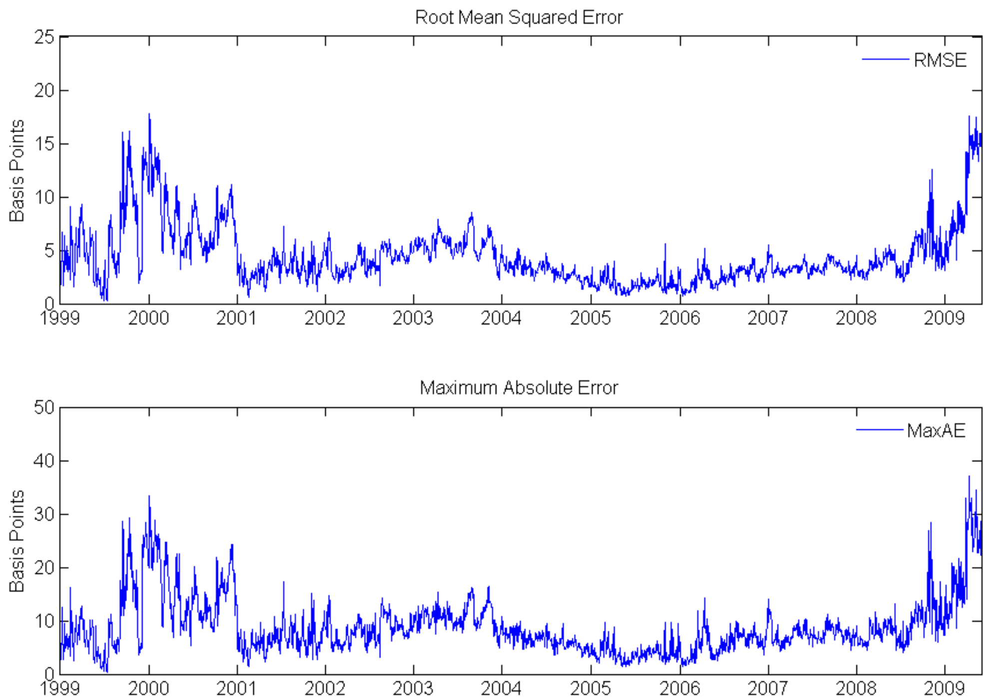


Figure 4: The top chart displays the Root Mean Squared Error (RMSE) for yields to maturity. The bottom chart displays the Maximum Absolute Error (MaxAE) for yields to maturity. Both errors are measured in basis points.

error measures over our entire sample are very plausible. The average RMSE is only 4.4 basis points. The highest value is 17.8 basis points on January 5, 2000. The average MaxAE is just 8.8 basis points. The highest value, i.e. the maximum absolute value of residua between the observed and the fitted yield to maturity, is 37.1 basis points for the 4.70%/22 bond (issue number 52) on April 8, 2009.

Figure 4 reveals that the fit substantially improved at the beginning of 2001. The switching from the Neslon-Siegel model to the Svensson model is not the only reason for the fit improvement. The Svensson model RMSE and MaxAE are very close to the Nelson-Siegel model RMSE and MaxAE for most days of 2000. I assume that the improvement occurred as the bond market became more developed and active, which decreased pricing anomalies across various bonds. I present the market activity in the top chart of Figure 5. The chart displays the monthly traded

that I consider the PRIBOR MID rate as the "fixing variable" of the very short end of the yield curve and thus request at least six bonds observations.

values in billions of Czech koruna of all government bonds used in the model estimation.[9] An increase in traded values since beginning of 2001 is clearly apparent from this chart. A simple linear regression of the daily RMSE values of the Svensson model on the daily traded values in period from July 3, 2000 till July 2, 2001 (251 observations) has the R^2 of 9.4% and the slope coefficient is equal to -0.38 (correct sign, p-value < 0.001). However, the relation between the RMSE and traded values is not, on different subsamples, always significant and the slope coefficient has even, on some subsamples, wrong sign.

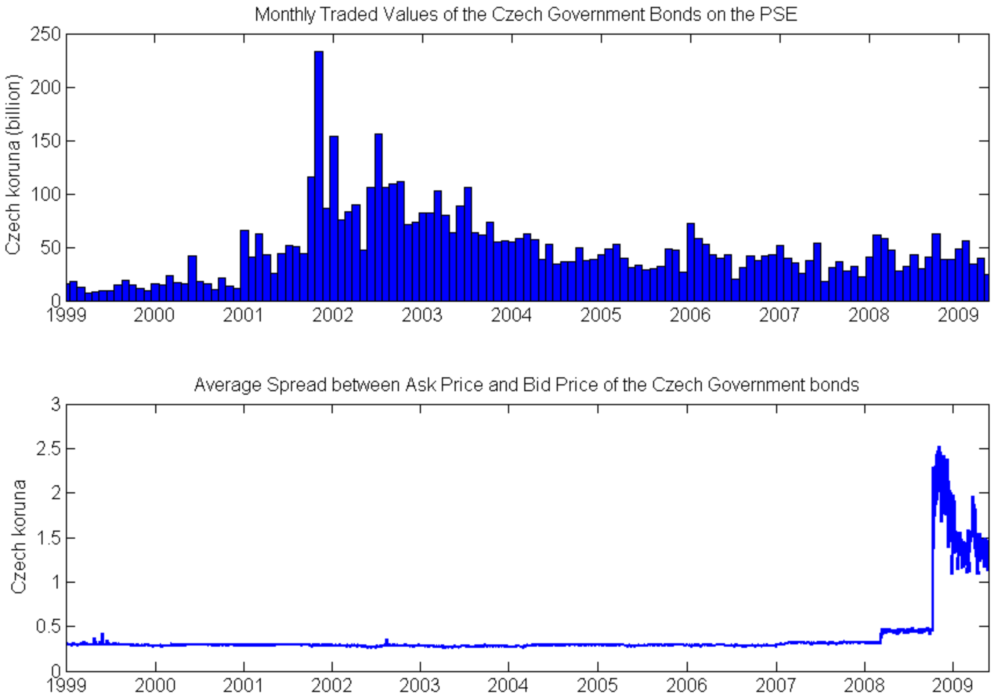


Figure 5: The top chart displays monthly traded values of government bonds in billions Czech koruna. The bottom chart displays average price spread between the ask and bid prices of government bonds. Both charts only consider government bonds used in the model estimation.

We can see in Figure 4 that the fit is generally very good from beginning of 2001 till the end of the third quarter of 2008. The fit has worsened again during the current financial market crisis, which started in 2008. The top chart of Figure 5 unveils that the traded values have not, surprisingly, decreased during the current financial crisis. However, the market has become more nervous, which is documented by the spread of bond price quotes shown in the bottom chart of Figure 5. This chart displays the daily average price spread between the ask and bid prices of our data set.[10] The average spread had been very stable around 30 haléře (standard deviation = 1.3 haléře) for 100 koruna notional principal from the beginning of our sampling period until March 2008 when it jumped to 42 haléře and had been oscillating around 45 haléře (standard deviation = 1.8 haléře) for the next 6 month. On November 8, 2008 the average price spread soared to 2.29 koruna and it has stayed very high, ranging from 1.08 koruna to 2.52 koruna (standard deviation = 38.1 haléře), until now. A linear regression of the daily RMSE of the Svensson model on the daily average price spread in period from January 1, 2009 till the last day of our data set, May 29, 2009 (351 observations) has the R^2 of 29.7% and the slope coefficient

[9] The traded values are provided by the Prague Stock Exchange and comprise vast majority of Czech government bonds trades.

[10] I calculate the average price spread for a given date as an unweighted arithmetic average of differences between ask and bid price quotes off all bonds that I use in estimation on that date.

is equal to 4.05 (correct sign, p-value < 0.001). The relation between the RMSE and the price spread is more robust, on different subsamples, compared to the relation between the RMSE and the traded values. The large difference between the ask and bid price quotes indicates the uncertainty of the market, which can turn into inconsistent pricing of bonds. We can expect that error measures will decline when the current crisis ends and the price spread will decrease on a reasonable level.

6.2 Individual Yield to Maturity Errors

I dig deeper into the estimation error and plot the evolution of yield to maturity errors for each bond used in estimation in Figure 9 in Appendix B. As we estimate the yield curve day by day and do not take into account the time series behavior of the yield curve, which is strongly persistent for all maturities, the estimation errors are strongly autocorrelated. It is desirable, that the errors oscillate around zero, i.e. they do not exhibit any systematic behavior.[11] Some bonds, especially at the beginnig of our estimation period, show relatively long intervals, when the estimation error stays constantly positive or negative (issue numbers 23, 25, 29, 36, 45, 49, 50 and 52). This is unwelcomed, but because of small number of bonds avaiable, I have decided not to exclude these bonds from the estimation. Again, it is apparent from Figure 9 that since the current financial market crisis, the errors have been rising for every outstanding bond (issue numbers: 33, 34, 36, 40, 41, 44, 45, 46, 48, 49, 50, 51, 52 and 54).

I do not present the price error measures, neither individual price errors in this paper, since they do not bring any new insight into the estimation results. I can provide them on a request.

6.3 Estimated Spot Rates

From the estimated parameters, I calculate spot rates and hence forward and par rates for each day of our data set. The estimated spot, forward and par rates are posted in data appendix that accompanies this paper. I provide a three-dimensional plot of the estimated spot rates in Figure 6. The three-dimensional plot presents the estimated spot yield curves, day after day, from January 4, 1999 until May 29, 2009. For each yield curve, the shortest maturity is one year and the longest maturity is given by the maximum maturity of the bond used in estimation, more precisely the maximum maturity of the bond is rounded towards the nearest integer towards infinity. Of course, it is possible to calculate spot rates for maturities beyond the maturity range used in estimation.

Further, I slice the three-dimensional plot to capture the time series of estimated spot rates for selected maturities in Figure 7. The blank space in the time series occurs when the maturity is not covered by the bonds used in estimation. The horizontal line represents the unconditional mean of the rates. Since 1999 the Czech spot rates exhibit downward trend with some upward spikes and approximately one year upward trend (from mid 2003 until mid 2004) before they reach their bottom levels in mid 2005. From mid 2005 till mid 2008, the rates revert back to their unconditional means. This supports the mean reverting property of interest rates – one of the most important stylized facts of interest rate behavior. It also gives some hope for the rejection of the unit root in interest rate time series, which is a fundamental requirement for basically any time series econometric modeling.

The slope, measured as 5-year rate minus 1-year rate, is positive since beginning of February 1999. The yield curve is inverted or rather flat for maturity range 1 to 5 years only in January 1999. Since the third quarter of 2008, the slope of the yield curve steepens substantially, as the 1-year rate quickly falls to low levels.

[11] Systematic overpricing was the reason for excluding the 6.08%/2001 bond (issue number 27) from the estimation, see Section 4.1.

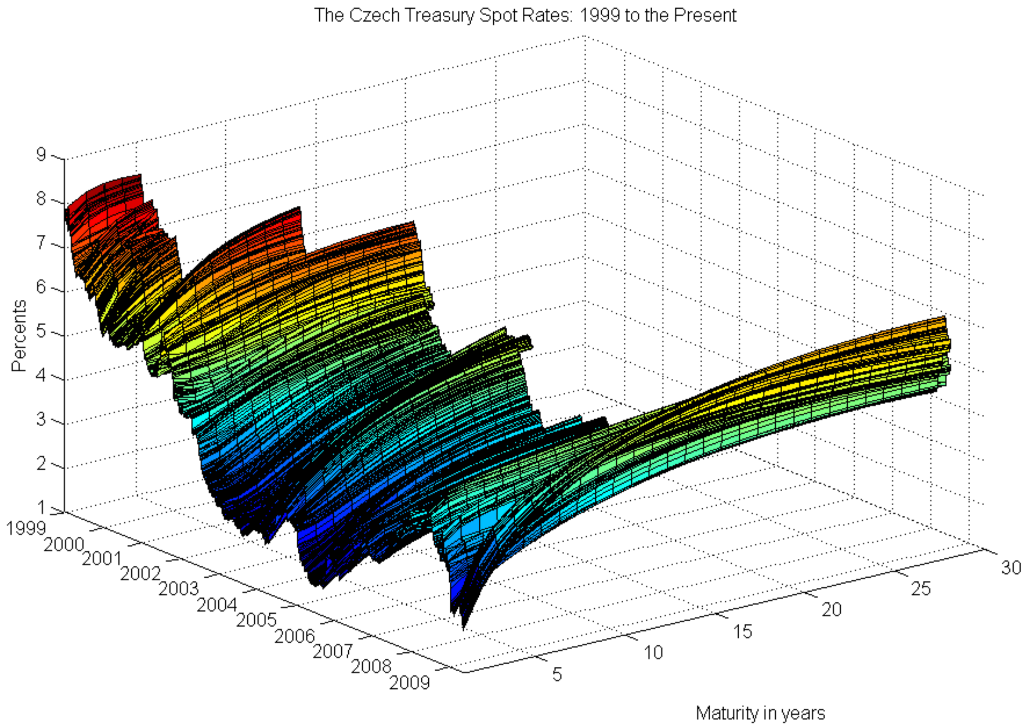


Figure 6: Estimated zero-coupon yield curves (continuously compounded). For each curve, the shortest maturity is one year and the longest maturity is given by the maximum maturity of the bond used in estimation.

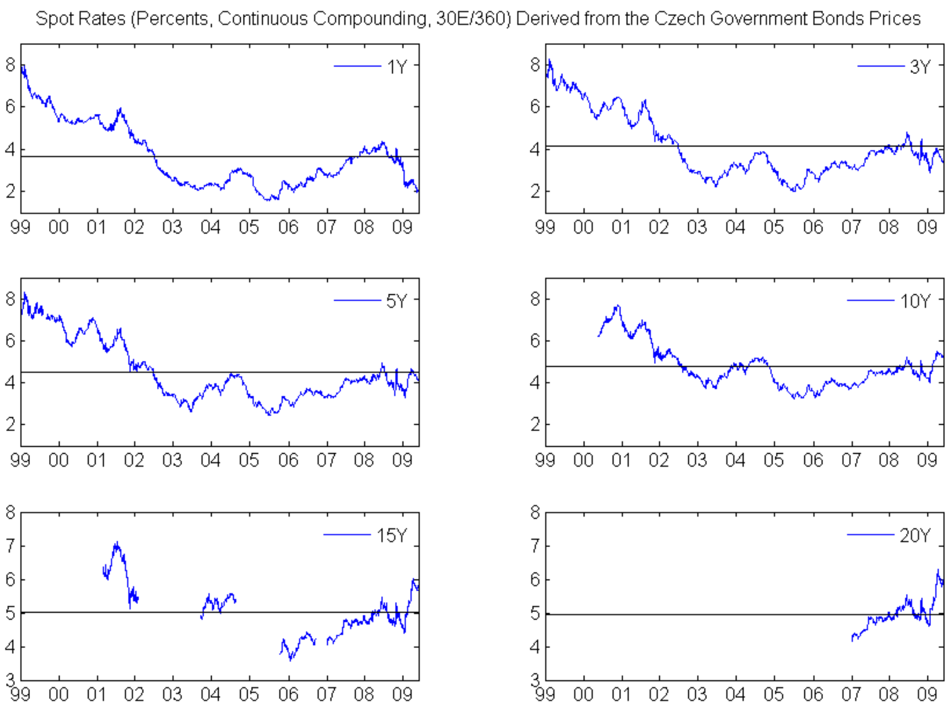


Figure 7: Estimated spot rates (continuously compounded) for selected maturities. The horizontal line represents the unconditional mean of the rates.

6.4 Estimated Par Rates versus Swap Rates

As I have mentioned in Section 2.2, interest rate swaps are an example of par rates quoted on the financial markets. The swap yield curve is often considered as another benchmark yield curve (besides the Treasury yield curve) of the economy. The swap rates and the Treasury rates can be compared to each other to measure the credit risk (possibility of default) of the country. A swap contract entails some credit risk, but potential losses from defaults on a swap are much less than the potential losses from defaults on a bond with the same notional principal. This is because the value of the swap is usually only a small fraction of the value of the bond.

I transform the estimated spot rates into discount functions according to (2) and then bootstrap using (9) to get the Treasury par rates. I use end of day swap rates quotes from Bloomberg. The swap rates and also PRIBOR rates use ACT/360 day count convention. Thus, I multiply the swap rate quotes by 365/360 fraction to approximately adjust them to the bonds 30E/360 day count convention. I plot the Treasury par rates versus swap rates and their spreads in Figure 8. The first two upper left hand side charts plot 3-year, respectively 10-year swap and

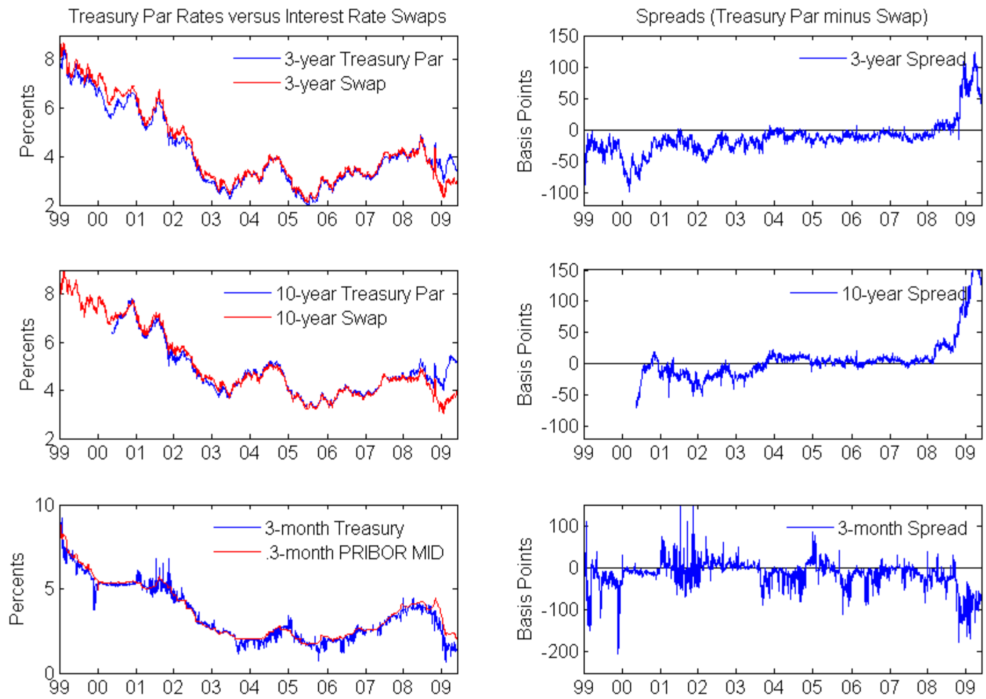


Figure 8: The first two left-hand side charts show 3-year and 10-year swap together with corresponding Treasury par rates. The bottom chart shows 3-month Treasury rate together with corresponding 3-month PRIBOR MID rate. All rates are annually compounded. The right-hand side charts provide detailed look at the corresponding spreads.

Treasury par rates. The bottom chart plots 3-month Treasury rate and 3-month PRIBOR MID rate. As the Treasury and swap rates time series copy each other very closely, I present the corresponding spreads in right hand side charts. The spreads, calculated as Treasury rate minus swap rate, document several issues.

First, the estimated Treasury par rates follow the dynamics of quoted swap rates very precisely for all maturities, which reconfirms the quality of estimated Treasury rates.

Second, the spreads dramatically rise during the current financial market crisis. The spreads already started to rise since March 2008. The average 3-year spread is minus 11 basis points from the beginning of March 2007 till the end of February 2008, whereas it is plus 29 basis points from the beginning of March 2008 till the end of February 2008 and it reaches maximum of 123 basis points on March 24, 2009. The widening spread opens a debate, which yield curve should be considered as the risk-free benchmark yield curve.[12] In my view, the Czech Treasury curve is the principal risk-free benchmark curve of the Czech economy. The reason is straightforward – interest rates swap is just an agreement to exchange payments, it is not an instrument for investing money. Money must be invested into some asset and Czech Treasury bonds have naturally the lowest credit risk among any investment instruments denominated in the Czech koruna.

Third, the spread between the 3-month Treasury rate and 3-month PRIBOR MID rate documents that the estimated 3-month Treasury rate has unrealistically high volatility, which is caused by using the PRIBOR MID overnight rate for fixing the very short end of the curve. Therefore, the estimated rates with a maturity under one year should not be considered as representative Treasury bond rates.

7 Conclusion

I have estimated the Czech Treasury yield curve from 1999 to the present. I use simple and parsimonious models of (Nelson and Siegel, 1987) and (Svensson, 1995). The models prove to fit the Czech government bonds price data very well for virtually each business day. I pay close attention to the estimation errors, which provide evidence of the estimation success.

Despite being a fundamental economical variable, the estimated Czech Treasury yield curve is not available. I fill this gap and create time series of spot and hence forward and par interest rates at a daily basis, extending back to the past as much as possible. I believe that the presented yield curve, which is a natural benchmark yield curve of the Czech economy, will initiate further research in empirical finance and applied macroeconomics.

The data appendix that accompanies this paper provides estimated spot rates (continuously compounded), instantaneous forward rates (continuously compounded) and par rates (coupon equivalent) at the daily frequency from January 4, 1999. The estimated parameters are also included in the data appendix and thus interest rates of practically any maturity can be calculated. I intend to periodically update the resulting yield curve data.

[12]For example, this issue is important for the Czech Society of Actuaries. The risk-free benchmark yield curve is used in the liability adequacy test. For ongoing discussion on this topic visit http://www.actuaria.cz.

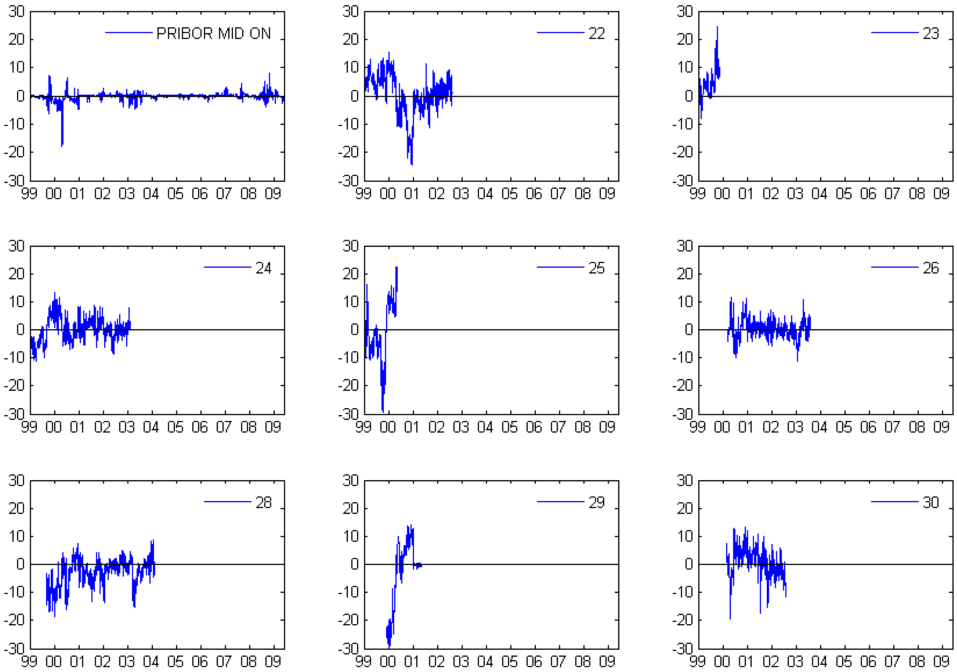
APPENDIX

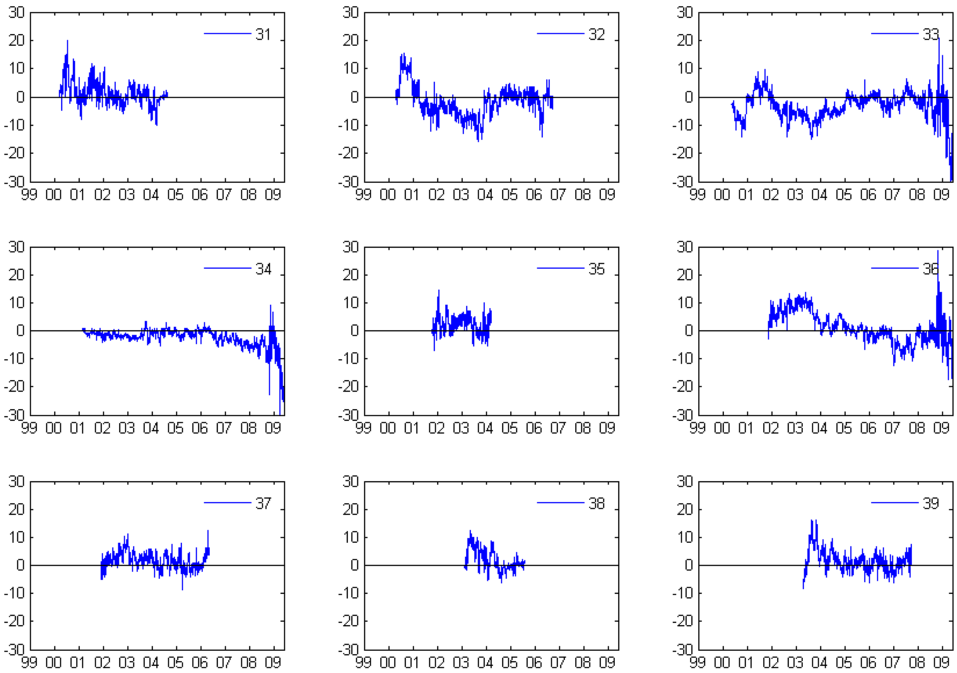
A Czech Government Bonds Used in Yield Curve Estimation

Table 2: This table lists all bonds used in estimation of the Czech Treasury yield curve from 1999 to the present. The issue number is a unique identifier which is incremented by one for every new issuance of the Czech government bond.

ISSIN	Issue Date	Maturity Date	Coupon Rate (%)	Issue Number
CZ0001002471	23/03/2009	11/04/2019	5.00	56
CZ0001002158	28/01/2008	11/04/2011	4.10	54
CZ0001001945	18/06/2007	12/09/2022	4.70	52
CZ0001001903	30/04/2007	11/04/2017	4.00	51
CZ0001001887	16/04/2007	18/10/2012	3.55	50
CZ0001001796	04/12/2006	04/12/2036	4.20	49
CZ0001001754	27/11/2006	27/11/2009	3.25	48
CZ0001001309	26/09/2005	26/09/2008	2.30	47
CZ0001001317	12/09/2005	12/09/2020	3.75	46
CZ0001001242	18/07/2005	18/10/2010	2.55	45
CZ0001001143	11/04/2005	11/04/2015	3.80	44
CZ0001000863	02/08/2004	02/08/2007	3.95	43
CZ0001000855	22/03/2004	22/03/2009	3.80	42
CZ0001000822	18/08/2003	18/08/2018	4.60	41
CZ0001000814	16/06/2003	16/06/2013	3.70	40
CZ0001000798	17/03/2003	17/03/2008	2.90	39
CZ0001000780	20/01/2003	20/01/2006	3.00	38
CZ0001000772	26/10/2001	26/10/2006	5.70	37
CZ0001000764	05/10/2001	05/10/2011	6.55	36
CZ0001000756	14/09/2001	14/09/2004	6.05	35
CZ0001000749	26/01/2001	26/01/2016	6.95	34
CZ0001000731	14/04/2000	14/04/2010	6.40	33
CZ0001000723	17/03/2000	17/03/2007	6.30	32
CZ0001000707	18/02/2000	18/02/2005	6.75	31
CZ0001000715	05/02/2000	05/02/2004	7.95	26
CZ0001000681	21/01/2000	21/01/2003	6.90	30
CZ0001000640	05/11/1999	05/11/2001	6.50	29
CZ0001000632	06/08/1999	06/08/2004	7.30	28
CZ0001000582	06/11/1998	06/11/2000	10.85	25
CZ0001000574	07/08/1998	07/08/2003	10.90	24
CZ0001000566	15/05/1998	15/05/2000	14.75	23
CZ0001000558	06/02/1998	06/02/2003	14.85	22

B Yield to Maturity Errors for Individual Bonds





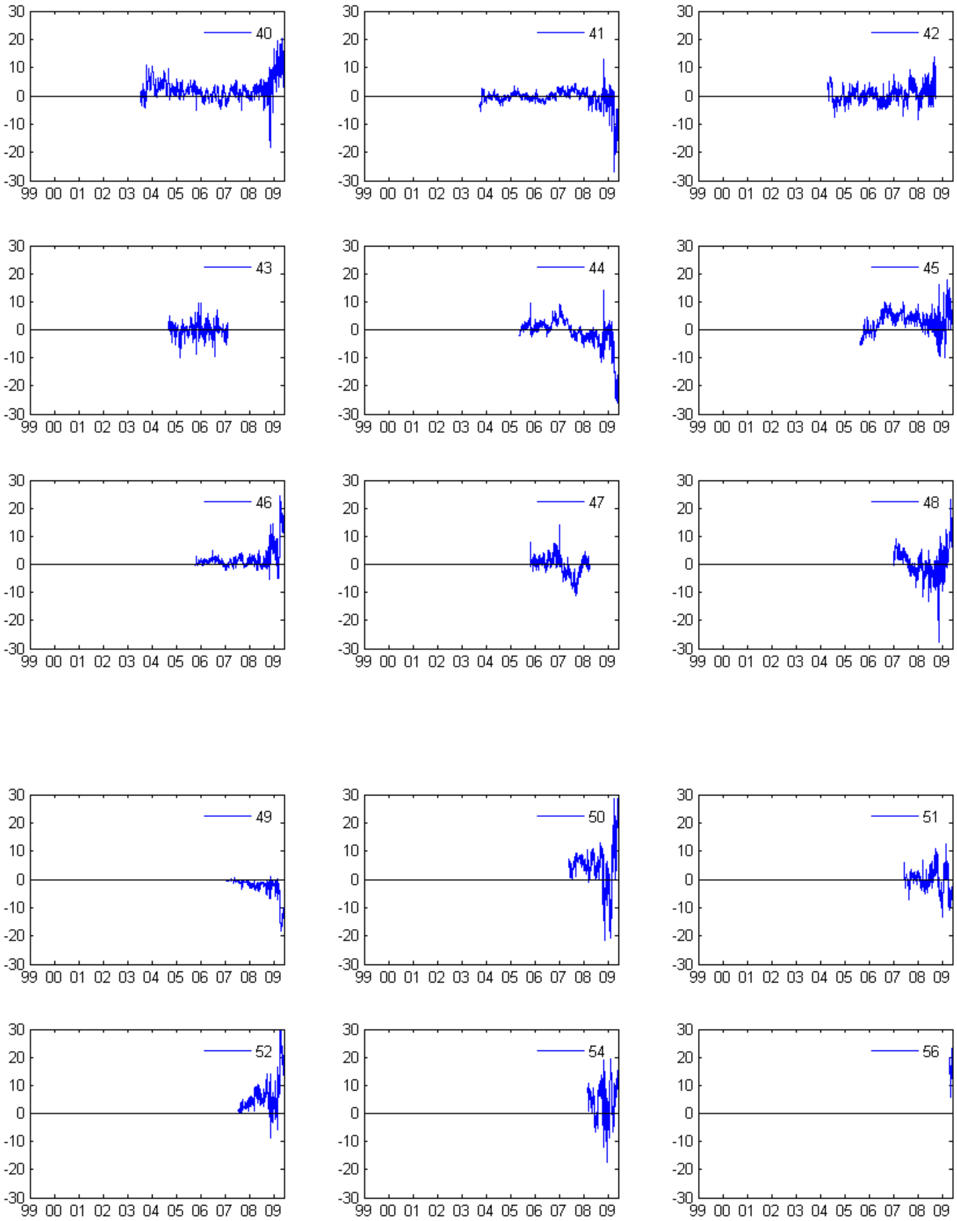


Figure 9: This figure displays yield to maturity errors in basis points, i.e. residuals between the observed and the fitted (model implied) yields to maturity. The bond can be identified by the issue number shown in legend.

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