Financial Frictions in a Small Open Economy: DSGE Model of the Czech Economy with Time-Varying Parameters

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Motivation

- Did any structural changes occur during recent financial and economic crisis?
- Which structural parameters did change and which are deep (time-invariant)?
- How was the behaviour of the economy affected by these structural changes?
- What was the relative importance of the structural changes compared to the structural shocks?
time-varying parameters are defined as unobserved states

\[ \theta_t = (1 - \alpha^\theta) \cdot \theta_{t-1} + \alpha^\theta \cdot \bar{\theta} + \nu^\theta_t \]

- \( \bar{\theta} \) is initial value of parameter \( \theta_t \)
- \( \alpha^\theta \) is an adhesion parameter
  - \( \alpha^\theta = 0 \Rightarrow \) random walk,
  - \( \alpha^\theta = 1 \Rightarrow \) white noise around \( \bar{\theta}_t \)
- \( \nu^\theta_t \sim N(0, \sigma^\theta_{\nu}) \)

\( \Rightarrow \) nonlinearity is introduced into the model \( \Rightarrow \) nonlinear state-space model

\[ x_t = g(x_{t-1}, w_{t-1}) \]

\[ y_t = h(x_t, v_t) \]
Non-linear filtering methods

- Kalman filter is optimal for linear systems
- Extended Kalman filter (Jacobian matrix of the state vector) can be used for nonlinear systems but performs poorly for severe nonlinearities

⇒ Nonlinear filters
  - with additive Gaussian noise - Extended Kalman filters
    - Monte Carlo based
    - Transformation based
  - with non Gaussian noise - Particle filters
    - Gaussian particle filter
    ⇒ Unscented particle filter
Unscented transformation

- method of calculating the statistics of a nonlinear transformation of random variable
- estimates are accurate up to the second order of Taylor expansion of the transformation function
- suppose that we have an $n$-dimensional random variable $x$ with mean $\bar{x}$ and covariance matrix $P_x$
- to calculate the statistics of its nonlinear transformation $y = f(x)$ we have to calculate a set of sigma points and weights $\{X_i, W_i\}_{i=0}^{2n}$
Unscented transformation, continued

\[ X_0 = \bar{x} \]
\[ W_0 = \frac{\kappa}{(n + \kappa)} \quad i = 0 \]
\[ X_i = \bar{x} + \left( \sqrt{(n + \kappa)P_x} \right)_i \]
\[ W_i = \frac{1}{2(n + \kappa)} \quad i = 1, \ldots, n \]
\[ X_i = \bar{x} - \left( \sqrt{(n + \kappa)P_x} \right)_{i-n} \]
\[ W_i = \frac{1}{2(n + \kappa)} \quad i = n + 1, \ldots, 2n \]

- \( \kappa \) is a scaling parameter
- \( \left( \sqrt{(n + \kappa)P_x} \right)_i \) is the \( i \)th column of the matrix square root
- mean and covariance matrix of \( y \) can then be described as

\[ \bar{y} = \sum_i W_i f(X_i) \]
\[ P_y = \sum_i W_i (f(X_i) - \bar{y})(f(X_i) - \bar{y})^T \]
Unscented particle filter

1 Initialization: set the prior mean $\bar{x}_0$ and covariance matrix $P_0$ for the state vector $x_t$.

2 Generating particles: Draw a total of $N$ particles $x_t^{(i)}$, $i = 1, \ldots, N$ from distribution $p(x_t)$ with mean $\bar{x}_t$ and covariance matrix $P_t$.

3 Unscented transformation: Calculate sigma points and weights $\{X_i, W_i\}_{i=0}^{2na}$ for the random vector $x^a_t = \begin{bmatrix} x_t & w_t & v_t \end{bmatrix}^T$.

4 Time Update: propagate each particle into future with the use of sigma points and transition and measurement equation and calculate means $\bar{x}_{t+1|t}^{(i)}$, $\bar{y}_{t+1|t}^{(i)}$ and covariance matrices $P_{t+1|t}^{(i)}$, $P_{y,y}^{(i)}$ and $P_{x,y}^{(i)}$. 
Unscented particle filter, continued

5 Unscented Kalman filter: For each particle calculate

\[ K_{t+1}^{(i)} = P_{x,y}^{(i)} \left( P_{y,y}^{(i)} \right)^{-1}, \]
\[ \bar{x}_{t+1}^{(i)} = \bar{x}_{t+1|t} + K_{t+1}^{(i)} (y_{t+1} - \bar{y}_{t+1|t}), \]
\[ P_{t+1}^{(i)} = P_{t+1|t} - K_{t+1}^{(i)} P_{y,y}^{(i)} \left( K_{t}^{(i)} \right)^{T}. \]

6 Weights update: for each particle, draw a sample \( x_{t+1}^{(i)} \) from

\[ q(x_{t+1}^{(i)} | x_0:t, y_1:t+1) = N(\bar{x}_{t+1}^{i}, P_{t+1}^{(i)}) \]
and evaluate the importance weight

\[ w_{t+1}^{i} \propto \frac{p(y_{t+1} | x_{t+1}^{i}) p(x_{t+1}^{i} | x_{t}^{(i)})}{q(x_{t+1}^{i} | x_0:t, y_1:t+1)}. \]

For all particles together, normalize the weights and calculate

\[ \bar{x}_{t+1} = \sum_{i} w_{t+1}^{(i)} x_{t+1}^{(i)}, \]
\[ P_{t+1} = \sum_{i} w_{t+1}^{(i)} (x_{t+1}^{(i)} - \bar{x}_{t+1})(x_{t+1}^{(i)} - \bar{x}_{t+1})^{T}. \]
12 runs of the UPF with 30,000 particles each were calculated for the second order approximation of the model.

Initial values of the time-varying parameters ($\bar{\theta}$) were set to the posterior means of the Bayesian estimation of the model with constant parameters.

Standard deviations of time-varying parameter innovations ($\sigma_\nu^\theta$) were set proportional to the standard deviations of posterior estimates.
Overall structure of the DSGE model of a small open economy (SOE) is based on Shaari (2008), who incorporated the financial accelerator mechanism á la Bernanke et al. (1999) into the basic SOE model of Galí and Monacelli (2005).

The model contains following optimizing representative agents: households, entrepreneurs and domestic and foreign retailers.

The monetary policy of the central bank is modelled with the use of forward looking Taylor rule.

Exogenous shock in entrepreneurial net worth is introduced into the model in this paper.
Data

- Quarterly time series of the period between 1996Q1 and 2012Q4
- Domestic economy: real aggregate product, consumer price index, 3-month PRIBOR and Prague stock exchange PX index as a proxy for the entrepreneurial net worth
- Foreign economy (EA12): real aggregate product, CPI index and 3-month EURIBOR
- CZK/EUR real exchange rate
- Original time series were transformed so as to express percentage deviations from steady state
Measurement errors

- Real output error ($y_{obs} - y$)
- Interest rate error ($r_{obs} - r$)
- CPI inflation error ($\pi_{obs} - \pi$)
- Foreign real output error ($y^*_{obs} - y^*$)
- Foreign interest rate error ($r^*_{obs} - r^*$)
- Foreign CPI inflation error ($\pi^*_{obs} - \pi^*$)
- Real exchange rate error ($rer_{obs} - rer$)
- Entreprise net worth error ($nw_{obs} - nw$)
Filtered external finance premium
Time-varying parameters

- $\Psi_B$ ... risk premium elast.
- $\Psi_I$ ... capital adj. costs
- $\gamma$ ... foreign goods preference bias
- $\chi$ ... ext. fin. premium elast.
- $\zeta$ ... bankruptcy rate
- $\Gamma$ ... capital/net worth ratio
- $\rho$ ... Taylor rule, smoothing
- $\beta_{\pi}$ ... Taylor rule, inflation
- $\Theta_y$ ... Taylor rule, output gap
Filtered shock innovations

- $\varepsilon_y$ ... productivity innovations
- $\varepsilon_{\text{LOP}}$ ... LOP innovations
- $\varepsilon_\pi^*$ ... foreign output innovations
- $\varepsilon_{\text{MP}}$ ... monetary policy innovations
- $\varepsilon_{\text{UIP}}$ ... UIP innovations
- $\varepsilon_{\text{NW}}$ ... survival rate innovations
- $\varepsilon_{\text{LOP}}$ ... LOP innovations
- $\varepsilon_\pi^*$ ... foreign inflation innovations
- $\varepsilon_\pi^*$ ... foreign interest rate innovations
- $\varepsilon_{\text{NW}}$ ... survival rate innovations
IRF of real output to filtered foreign demand shock
IRF of real output to constant foreign demand shock
IRF of net worth to filtered foreign demand shock
IRF of net worth to constant foreign demand shock
IRF of real output to filtered exogenous shocks - comparison

- IRF of real output to UIP shock
- IRF of real output to LOP shock
- IRF of real output to foreign interest rate shock
- IRF of real output to foreign inflation shock
- IRF of real output to foreign output shock
- IRF of real output to net worth shock
- IRF of real output to domestic productivity shock
- IRF of real output to monetary policy shock
Time-varying parameters (SK)

- **$\Psi_B$**... risk premium elast.
- **$\Psi_I$**... capital adj. costs
- **$\gamma$**... foreign goods preference bias
- **$\Gamma$**... capital/net worth ratio
- **$\rho$**... Taylor rule, smoothing
- **$\beta_\pi$**... Taylor rule, inflation
- **$\Theta_y$**... Taylor rule, output gap
Time-varying parameters (EA)

- \( \Psi_B \) ... risk premium elast.
- \( \Psi_I \) ... capital adj. costs
- \( \gamma \) ... foreign goods preference bias
- \( \chi \) ... ext. fin. premium elast.
- \( \zeta \) ... bankruptcy rate
- \( \Gamma \) ... capital/net worth ratio
- \( \rho \) ... Taylor rule, smoothing
- \( \beta_\pi \) ... Taylor rule, inflation
- \( \Theta_y \) ... Taylor rule, output gap
Unscented particle filter was used to estimate a NL DSGE SOE model with financial frictions and time-varying parameters.

Results of the estimation suggest that some structural changes occurred during recent financial and economic crisis - especially in the financial sector.

Some parameters stayed relatively stable and can be therefore considered deep (habit in consumption, Calvo parameters).

Behaviour of model economy was affected by the changing structure to some extent.

However, it was probably the exogenous shocks that played the dominant role during the crisis.

Further research will be directed at SOE in monetary union (SK) and large open economy (EA).
References


Thank you for your attention!

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