Testing non-linearity in financial returns using a modified Q test

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Introduction

- Testing for non-linearity is of much practical importance for applied time series analysis in economics/finance.
- One of the most widely used tests is a portmanteau test (MLQ) proposed by McLeod and Li (1983).
- However, Vávra (2012) shows that the MLQ test suffers from the following shortcomings:
  1. The MLQ test cannot detect some interesting non-linear processes (e.g. NLMA);
  2. The MLQ test does exhibit a low power against standard non-linear processes (e.g. TAR, TMA);
  3. The MLQ test lacks a discrimination power against advanced GARCH processes (e.g. asymmetric GARCH)

The main task of the paper

The main task is to propose a new version of the Q test based on generalized correlations to fix the aforementioned shortcomings of the MLQ test.

Assumptions

Assumption 1 Let us assume $\{X_t : t \in \mathbb{Z}\}$ is a zero-mean real-valued finite-order ARMA(p,q) model given by

$$X_t = \xi_1 X_{t-1} + \cdots + \xi_p X_{t-p} + \zeta_1 a_{t-1} + \cdots + \zeta_q a_{t-q} + a_t, \qquad (1)$$

where $\{a_t : t \in \mathbb{Z}\}$ is a sequence of IID(0,σ^2) model innovations such that $\mathbb{E}(|a_t|^8) < \infty$. Let $\boldsymbol{\beta} = (\xi_1, \ldots, \xi_p, \zeta_1, \ldots, \zeta_q, \sigma)'$ be a $(p+q+1 \times 1)$ parameter vector, which is assumed to be in the interior of the parameter space

$$\begin{aligned}\mathbf{B} = \{\boldsymbol{\beta} \in \mathbb{R}^{p+q} \times \mathbb{R}_{++} : \; & \xi(z) = 1 - \sum_{i=1}^{p} \xi_i z^i \neq 0 \quad \text{for all} \quad |z| \leq 1, \\ & \zeta(z) = 1 - \sum_{i=1}^{q} \zeta_i z^i \neq 0 \quad \text{for all} \quad |z| \leq 1, \\ & \xi(z) \quad \text{and} \quad \zeta(z) \quad \text{have no root in common}\}.\end{aligned}$$

$\square$

Modified Q tests

The generalized correlation is defined as follows

$$\rho_{rs}(k) = \frac{\gamma_{rs}(k)}{\gamma_{rs}(0)} = \frac{\mathbb{E}\left[g_r(a_t)g_s(a_{t-k})\right]}{\sqrt{\mathbb{E}\left[g_r^2(a_t)\right]\mathbb{E}\left[g_s^2(a_t)\right]}}, \tag{2}$$

where $g_r(\cdot)$ and $g_s(\cdot)$ are assumed to be real-valued zero-mean continuous functions given by

$$g_r(a) = a^r - \mathbb{E}(a^r), \quad g_s(a) = a^s - \mathbb{E}(a^s), \quad \text{for} \quad r, s \in \{1, 2\}. \tag{3}$$

Example

For $r = 2$ and $s = 1$, $\gamma_{21}(k) = \mathbb{E}\left[(a_t^2 - \mathbb{E}(a_t^2))(a_{t-k} - \mathbb{E}(a_{t-k}))\right]$ for some integer $k > 0$.

The null hypothesis of linearity

- $H_0 : \forall_k \; \rho_{rs}(k) = 0$ for $(r, s) \in \{(1, 2), (2, 1), (2, 2)\}$,
- $H_1 : \exists_k \; \rho_{rs}(k) \neq 0$ for $(r, s) \in \{(1, 2), (2, 1), (2, 2)\}$.

Modified Q tests

The generalized Q test is then defined as follows

$$Q_{rs}(m) = \sum_{k=1}^{m} (T-k)\hat{z}_{rs}(k)^2, \tag{4}$$

where $\hat{z}_{rs}(k)$ denotes the transformed generalized correlation coefficient given by

$$\hat{z}_{rs}(k) = \frac{1}{2}\log\left(\frac{1+\hat{\rho}_{rs}(k)}{1-\hat{\rho}_{rs}(k)}\right). \tag{5}$$

Fact

Note that for $r = s = 2$, the Q_{22} test is an anlogy of the MLQ test proposed by McLeod and Li (1983).

Asymptotic results

Theorem 1 Under Assumption 1, the limiting distribution of a vector of sample generalized correlations $\hat{\mathbf{z}}_{rs} = (\hat{z}_{rs}(1), \ldots, \hat{z}_{rs}(m))'$ is given by

$$\sqrt{T}(\hat{\mathbf{z}}_{rs} - \mathbf{z}_{rs}) \xrightarrow{d} N(\mathbf{0}, \mathbf{I}),$$

for integers $(r, s) \in \{(1,2), (2,1), (2,2)\}$ and some integer $m > 0$. $\square$

Proof. See Vávra and Psaradakis (2013, p. 21).

Theorem 2 Under Assumption 1, the limiting distribution of the Q tests is given by $Q_{rs}(m) \xrightarrow{d} \chi^2(m)$, for integers $(r, s) \in \{(1,2), (2,1), (2,2)\}$ and some integer $m > 0$. $\square$

Proof. See Vávra and Psaradakis (2013, p. 22).

Monte Carlo setup

- The size properties of the Q tests are tested using 2 linear models: AR(#8), MA(#8).
- The size properties of the Q tests are tested using 7 non-linear models: TAR(#24), TMA(#24), GARCH(#17), NLGARCH(#17), BL(#18), NLMA(#12), MSAR(#24).
  2 samples sizes: $T \in \{200, 1000\}$.
- Number of Monte Carlo repetitions: $R = 1000$.
- Model innovations: $a \sim N(0, 1)$.
- Lag order configuration: $m \in \{5, 10, 15\}$ and $\hat{m}$ denotes the automatic lag selection based on Escanciano and Lobato (2009).

Monte Carlo results: size properties

test	lag m	T=200		T=1000	
		AR	MA	AR	MA
Q_{12}	5	0.050	0.049	0.049	0.050
	10	0.046	0.054	0.048	0.050
	15	0.045	0.049	0.051	0.050
	$\hat{m}$	0.050	0.049	0.053	0.054
Q_{21}	5	0.049	0.049	0.049	0.050
	10	0.051	0.044	0.050	0.050
	15	0.048	0.049	0.048	0.051
	$\hat{m}$	0.049	0.047	0.052	0.048
Q_{22}	5	0.049	0.054	0.052	0.050
($\approx MLQ$)	10	0.054	0.058	0.051	0.051
	15	0.056	0.058	0.049	0.054
	$\hat{m}$	0.057	0.055	0.054	0.057

[a] m denotes the automatically selected lag order based on Escanciano and Lobato (2009).

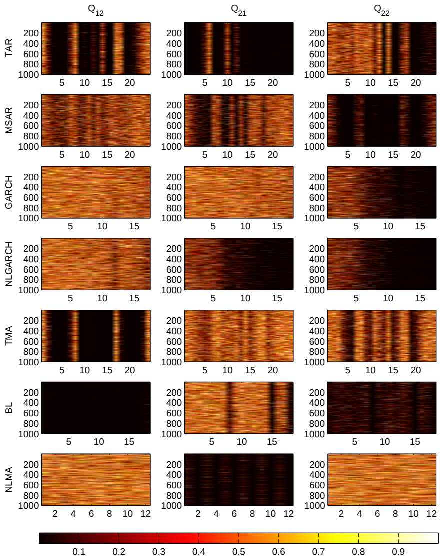
[b] The significance level is set to $\alpha = 0.05$.

Monte Carlo results: power properties

test	lag m	T=1000						
		TAR	TMA	BL	NLMA	GARCH	NLGARCH	MSAR
Q_{12}	5	0.65	**0.78**	**1.00**	0.05	0.08	0.11	0.26
	10	0.61	**0.76**	**1.00**	0.05	0.08	0.11	0.21
	15	0.59	**0.74**	**1.00**	0.05	0.08	0.11	0.19
	$\hat{m}$	0.64	**0.79**	**1.00**	0.06	0.09	0.13	0.27
Q_{21}	5	**0.89**	0.08	0.19	**0.77**	0.09	**0.68**	0.41
	10	**0.87**	0.07	0.18	**0.69**	0.08	**0.63**	0.36
	15	**0.86**	0.07	0.17	**0.65**	0.08	**0.58**	0.33
	$\hat{m}$	**0.89**	0.13	0.19	**0.89**	0.09	**0.69**	0.41
Q_{22}	5	0.44	0.19	0.69	0.05	0.63	0.75	0.84
($\approx MLQ$)	10	0.42	0.15	0.60	0.05	0.58	0.70	0.80
	15	0.40	0.13	0.55	0.05	0.54	0.67	0.78
	$\hat{m}$	0.45	0.28	0.77	0.06	0.66	0.77	0.84

[a] m denotes the automatically selected lag order based on Escanciano and Lobato (2009).
[b] The significance level is set to $\alpha = 0.05$.

Monte Carlo results: sensitivity analysis



Empirical results

- 22 asset returns: 5 exchange rate series; 5 interest rate series; 6 commodity series; and finally 6 equity series;
- Average weakly returns spanning the period 1980 and 2010 (1600 observations)

Question

Is a simple GARCH model an adequate model for asset returns for a given data set?

Empirical results

variable/lag	Q_{12} 5	10	15	$\hat{m}$	Q_{21} 5	10	15	$\hat{m}$	Q_{22} 5	10	15	$\hat{m}$
Exchange rates												
USDGBP	0.37	0.46	0.43	0.12	0.08	0.11	0.03	0.03	0.00	0.00	0.00	0.00
USDJPY	0.97	0.94	0.66	0.00	0.10	0.03	0.07	0.21	0.00	0.00	0.00	0.00
USDCAD	0.37	0.66	0.80	0.31	0.89	0.92	0.96	0.96	0.00	0.00	0.00	0.00
USDAUD	0.88	0.70	0.81	0.48	0.05	0.19	0.24	0.33	0.00	0.00	0.00	0.00
USDCHF	0.85	0.99	1.00	0.98	0.43	0.25	0.24	0.25	0.00	0.00	0.00	0.00
Interest rates												
USIR3M	0.62	0.93	0.83	0.89	0.99	1.00	0.89	0.97	0.00	0.00	0.00	0.00
UKIR3M	0.11	0.08	0.02	0.01	0.62	0.53	0.80	0.86	0.00	0.00	0.00	0.00
AUIR3M	0.74	0.47	0.72	0.89	0.90	0.84	0.97	0.95	0.00	0.00	0.00	0.00
CAIR3M	0.13	0.01	0.01	0.05	0.83	0.34	0.35	0.48	0.00	0.00	0.00	0.00
CHIR3M	0.88	0.93	0.97	0.98	0.31	0.28	0.50	0.82	0.00	0.00	0.00	0.00
Equity indices												
DJIA	0.53	0.50	0.12	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FTSE	0.10	0.24	0.13	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AUSE	0.49	0.16	0.13	0.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TSE	0.66	0.69	0.50	0.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TOPIX	0.17	0.02	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CHSE	0.59	0.70	0.59	0.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

[a] m denotes the automatically selected lag order based on Escanciano and Lobato (2009).

[b] The estimated p-values of the Q tests are presented in the table.

Conclusion

- The proposed Q tests have very good size and power properties, the limiting χ^2 distribution is a very good approximation, regardless of the sample size T and the lag order m.
- The Q tests are capable to capture some interesting non-linear models, for which the original MLQ test completely fails (e.g. a NLMA model).
- The Q tests significantly improve the power against some non-linear models (e.g. TAR and TMA), for which the original MLQ test does not work very well.
- The Q tests can be used for discrimination between simple and more complicated (non-linear/asymmetric) GARCH models.
- The empirical results suggest that, although the null hypothesis of linearity is rejected by the Q_{22} test in all 22 cases, the results are not compatible with a simple GARCH model in almost 65 %. Our results have serious implications for value-at-risk modelling, among other things.

Thanks

Thank you for attention.

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