Dynamic Factor Models in Real Time: A New Approach and an Implementation in Matlab

Jan Brůha

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Outline of the presentation

1. Motivation
2. My contribution
3. Implementation in Matlab
   - Simulation Study

Motivation

Stock and Watson (2011)

The premise of dynamic factor models (DFM) is that a few latent dynamic factors f_t drive the comovements of high-dimensional vector of time-series variables X_t, which is also affected by a vector of disturbances ε_t:

$$X_t = \underbrace{\sum_i \Lambda_i f_{t-i}}_{\equiv \chi_t = \text{ common component}} + \underbrace{\varepsilon_t}_{\text{idiosyncratic part}} .$$

Applications

Dynamic factor models have a wide range of applications:

- Near-term forecasting
- Analysis of macroeconomic comovements
- Analysis of commodity markets
- Finance: yield curve
- and more ....

Stock and Watson (2011) distinguish several generations of DFM

First generation

First generation DFM have been estimated in time-domain using MLE

$$X_t = \Lambda_0 f_t + e_t$$

$$f_t = \Psi_1 f_{t-1} + \dots \Psi_j f_{t-j} + G\eta_t,$$

By the **parametric** specification of distribution for η_t (Gaussian iid errors) and e_t (usually Gaussian stationary process), the model can be converted to a state space form and the likelihood function evaluated using the Kalman filter:

- static nature:
  - no lead-lag relation among X_t
  - not always plausible, e.g. the unemployment cycles lags the output cycles in most advanced countries (Brůha & Polanský, 2014; Andrle, Brůha, Solmaz 2013, 2014).

Second generations /a

Non-parametric 'averaging' errors:

- (Generalized) principal component estimation
- $\min_{f_1,\ldots,f_T,\Lambda} T^{-1}\sum_{t=1}^{T}(X_t-\Lambda f_t)\Sigma_e^{-1}(X_t-\Lambda f_t)'$,
- Leads to the eigenvalue decomposition of $\Sigma_e^{-1/2}\widehat{\Sigma_x}\Sigma_e^{-1/2}$,
- Weak assumptions on dgp for f_t
- Standard PCA if Σ_e is replaced by the identity matrix (Bai & Ng, 2002, 2003)
- Hard to estimate precisely Σ_e
  - Forni, Halli, Lippi and Reichlin (2005) suggest estimating it by non-parametric spectral density (a dynamic model)
- Again **static** model, even if Σ_e estimated from a dynamic model

Second generations /b

Dynamic principal component analysis:

- Based on the frequency-domain PCA by Brillinger (1964)
- Introduced to empirical economics by Forni, Halli, Lippi and Reichlin (2000)
- A non-parametric method:
  - weak assumption on data generating process (rational spectral density),
  - the spectral density can be estimated by a non-parametric (Bartlett) approach.
- The spectral density is subjected to PCA and then transformed back to time-domain filter using a two-sided linear filter

Second generations /c

The transformation to time domain leads to a two-sided filter:

$$\chi_t = \sum_{i=-K}^{K} \omega_k X_{t-k},$$

where χ_t is the common component (driven by the common factors), and ω_k are filtered weights.

Implications:

- Forni, Halli, Lippi and Reichlin (2000) call it 'two-sided' DFM
  - versus the 'static' one-sided DFM based on generalized PCA by Forni, Halli, Lippi and Reichlin (2005) outlined above
- The common component χ_t cannot be estimated at the beginning and at the end of the sample
- Problems for many applications (real-time data)
- **My contribution: how to estimate the common component on the whole sample.**

Third generation – state space models

They can be applied to a variety of **parametric** models using sophisticated tools of modern econometrics, such as

- the EM algorithm (Bańbura & Modugno, 2010),
- the Gibbs sampler (Bai & Wang, 2012),
- the two-step estimation (Doz, Giannone and Reichlin, 2006).

These models:

- are truly dynamic (unlike the first generations and one-sided PCA),
- can accommodate various issues (missing data, asynchronous data release, ...),
- the common component estimated on the *whole* sample,
- but **are parametric**.

Quest

Quest

- Is it possible to propose:
  - a *genuinely* dynamic model,
  - based on non-parametric (*frequency-domain*) approach,
  - with the common component estimable on *the whole sample*,
  - and possibly accommodating *other issues*, such as asynchronous data releases?

Yes

... and this is my contribution.

How is it done?

Start with the common component representation:

$$X_t = \underbrace{\chi_t}_{\equiv \sum_i \Lambda_i f_{t-i}} + \varepsilon_t,$$

Now consider the linear projection of χ_t on $X_{\mathcal{S}} \equiv \{X_s\}_{s \in \mathbb{S}}$, where $\mathbb{S}$ is an *arbitrary* index set. The linear projection is given as:

$$\mathbb{E}^* [\chi_t | X_{\mathcal{S}}] = \mathbb{E}\left[\chi_t X_{\mathcal{S}}^T\right] \mathbb{E}\left[X_{\mathcal{S}} X_{\mathcal{S}}^T\right]^{-1} X_{\mathcal{S}},$$

Matrices such as $\mathbb{E}\left[\chi_t X_{\mathcal{S}}^T\right]$ can be derived from the spectral density of the data:

- as in 'second-generation' DFM (DPCA)
- hence, frequency-domain DPCA meets the linear regression.

How to get the common component on the whole sample?

The adaptation of the index set S:

- The index set can be adapted to the beginning and end of the sample (hence, real time data).
  - assume you wish to get $\mathbb{E}^* [\chi_t | \{X_{t-1}, X_t, X_{t+1}\}]$
  - trivial in the middle of the sample,
  - replace it by $\mathbb{E}^* [\chi_T | \{X_{T-1}, X_T\}]$ at the end of the sample!
- The index set can be adapted for missing data (e.g. due to asynchronous data release).

Finite-sample properties

The estimation of covariance matrices $\mathbb{E}\left[\chi_t X_{\mathcal{S}}^T\right]$ and $\mathbb{E}\left[X_{\mathcal{S}} X_{\mathcal{S}}^T\right]$ can be imprecise in finite samples.

How to overcome this complication:

- Use robust regression in computing the projection $\mathbb{E}^*\left[\chi_t | X_{\mathcal{S}}\right]$!
- I use a ridge regression with good properties.

Matlab codes

I build a set of Matlab routines that can be used to run a set of non-parametric DFM

- **sfactor.m** computes the static PCA
- **dfactor.m** computes the 'two-sided' version of the DPCA
  - for particular choices, equivalent to the 'one-sided' DFM representation,
- **tfactor.m** implements the approach outlined in this presentation.

I plan to put these routines on Matlab Exchange File soon

A simulation study /1

Forni, Halli, Lippi and Reichlin (2005) proposed four computational experiments with DFM.

M1 read as

$$x_{it} = \lambda_i f_t + \alpha c_i \epsilon_{it} \text{ with } (1 - .5L) f_t = u_t,$$

and the shocks ϵ_{it}, u_t, λ_i are independent standard normal variables, $c_i \sim \mathcal{U}_{[0.1\ 1.1]}$ and α is calibrated so that the average idiosyncratic-common variance ratio is 1.

M2 reads as:

$$x_{it} = \sum_{k=0}^{3} a_{ik} u_{1,t-k} + \sum_{k=0}^{3} b_{ik} u_{2,t-k} + \alpha c_i \epsilon_{it},$$

where a_{ik}, b_{ik} and the shocks are standard normal variables and c_i and α are as above.

A simulation study /2

M3 reads as:

$$x_{it} = \sum_{k=l_i}^{l_i+2} \lambda_{k-l_i,i} f_{t-k} + \xi_{it},$$

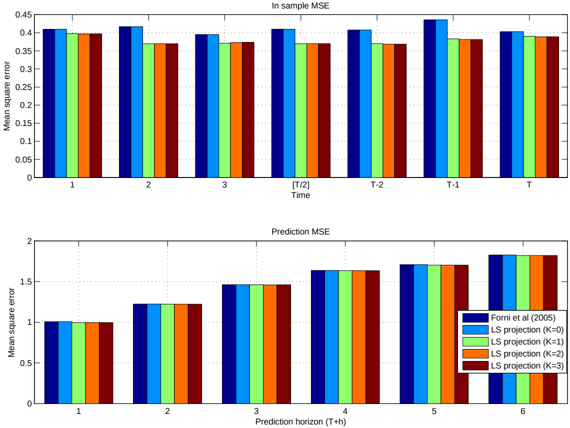
where $(1 - .5L)f_t = u_t$, $\xi_{it} = \alpha c_i(\varepsilon_{it} + \varepsilon_{i+1t})$, u_t and ε_{it} are independent standardized normal random variables,

$$l_i = \begin{cases} 0 & \text{for} \quad i \leq m \\ 1 & \text{for} \quad i \in \{m+1, \ldots, 2m\} \\ 2 & \text{for} \quad i \geq 2m+1 \end{cases}$$

M4 is the same as **M3**, but without cross-sectional dependence ($\xi_{it} = \alpha c_i \varepsilon_{it}$).

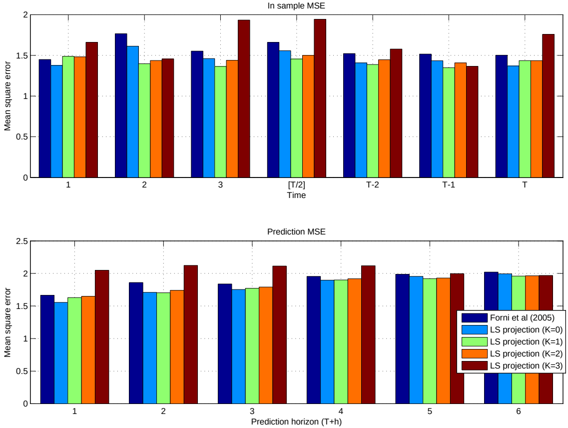
Model M1 – known covariance matrices

Marginal efficiency gains for this model



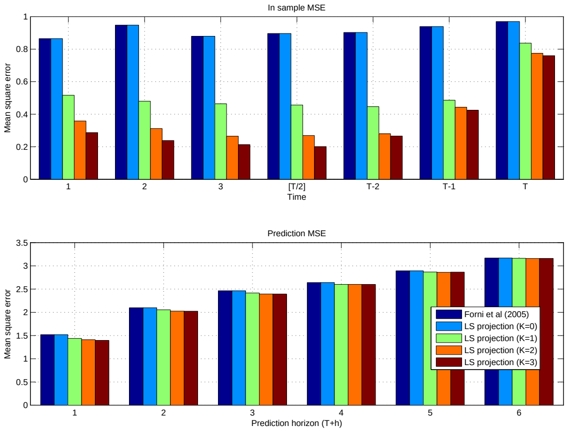
Model M1 – unknown covariance matrices

Gains disappear if covariance matrices are to be estimated



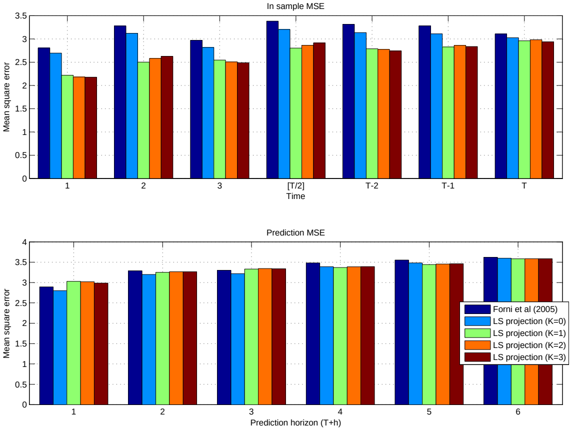
Model M3 – known covariance matrices

Large efficiency gains at the beginning of the sample



Model M3 – unknown covariance matrices

The efficiency gains survive even in small samples (T=50, N=20)



Disclaimer

The tool presented here was partly developed under the CNB research project B1/10. Nevertheless, the views expressed here are mine and do not necessarily reflect the position of the Czech National Bank.

I thank to Michal Andrle for comments and encouragement. However, any errors are solely my own responsibility.

Concluding slide

Thank you for your attention.

jan.bruha@cnb.cz