Nonlinear DSGE Model of the Czech Economy with Time-varying Parameters

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Motivation

- Did any structural changes occur during the turbulent period of recent financial and economic crisis of 2008–2009?

- Which structural parameters did change? Were the changes temporary or permanent?

- How was the behaviour of the economy affected by these structural changes?

- Which changes are specific for the Czech economy and which correspond to the europe-wide trends?
Time-varying parameters within state-space models

- Time-varying parameters are defined as unobserved states
  \[ \theta_t = (1 - \alpha_t^\theta) \cdot \theta_{t-1} + \alpha_t^\theta \cdot \bar{\theta} + \nu_t^\theta \]

- \( \bar{\theta} \) is initial value of parameter \( \theta_t \)
- \( \alpha_t^\theta \) is a time-varying adhesion parameter (panel)
  - \( \alpha^\theta = 0 \Rightarrow \) random walk,
  - \( \alpha^\theta = 1 \Rightarrow \) white noise around \( \bar{\theta}_t \),
  - \( \alpha^\theta = 0.25 \Rightarrow \) our choice
- \( \nu_t^\theta \sim N(0, \sigma_{\nu}^\theta) \)

\( \Rightarrow \) nonlinearity is introduced into the model \( \Rightarrow \) nonlinear state-space model

\[ x_t = g(x_{t-1}, w_{t-1}) \]

\[ y_t = h(x_t, \nu_t) \]
Non-linear filtering methods

- Kalman filter is optimal for linear systems
- Extended Kalman filter (Jacobian matrix of the state vector) can be used for nonlinear systems but performs poorly for severe nonlinearities

⇒ Nonlinear filters
  - with additive Gaussian noise - Extended Kalman filters
    - Monte Carlo based
    - Transformation based
  - with non Gaussian noise - Particle filters
    - Gaussian particle filter
    - Unscented particle filter
Non-linear particle filter

1 Initialization: $t = 0$, set the prior mean $\bar{x}_0$ and covariance matrix $P_0$ for the state vector $x_t$.

2 Generating particles: Draw a total of $N$ particles $x_{t+1}^{(i)}$, $i = 1, \ldots, N$ from distribution $p(x_{t+1})$ with mean $\bar{x}_{t+1|t}$ and covariance matrix $P_{t+1|t}$ (transition equation). Calculate $\bar{y}_{t+1|t}$ (measurement equation) and covariance matrices $P_{y,y}$ and $P_{x,y}$.

3 Kalman filter:

$$K_{t+1} = P_{x,y} (P_{y,y})^{-1},$$
$$\bar{x}_{t+1} = \bar{x}_{t+1|t} + K_{t+1}(y_{t+1} - \bar{y}_{t+1|t}),$$
$$P_{t+1} = P_{t+1|t} - K_{t+1}P_{y,y}(K_{t})^T$$

4 Time Update: $t = t + 1$. 

Non-linear particle filter, diagram
Non-linear particle filter setting

- 20 runs of the UPF with 30,000 particles each were calculated for the second order approximation of the model.
- Initial values of the time-varying parameters ($\bar{\theta}$) were set to the posterior means of the Bayesian estimation of the model with constant parameters.
- Standard deviations of time-varying parameter innovations ($\sigma^\theta$) were set proportional to the estimated posterior means (10%).
- Bayesian Random Walk Metropolis-Hastings estimation: two chains of 1,000,000 draws each, 50% burn-in sample, acceptance rate near 30%.
Model

- Overall structure of the DSGE model of a small open economy (SOE) is based on Shaari (2008), who incorporated the financial accelerator mechanism à la Bernanke et al. (1999) into the basic SOE model of Galí and Monacelli (2005).

- The model contains following optimizing representative agents: households, entrepreneurs and domestic and foreign retailers.

- The monetary policy of the central bank is modelled with the use of forward looking Taylor rule.

- Foreign sector observables are modelled as SVAR(1) block.
The model is estimated on two sets of data: CZ+EA (blue) and EA+US (red).

Quarterly time series of the period between 1999Q2 and 2013Q4, 59 observations.

Domestic economy: real aggregate product, real investment, consumer price index, 3-month PRIBOR (3-month EURIBOR).

Foreign economy EA17 (US): real aggregate product, CPI index and 3-month EURIBOR (3-month T-Bill rate).

CZK/EUR (EUR/USD) real exchange rate.

Original time series were transformed so as to express percentage deviations from steady state (HP filter, $\lambda = 1600$).
Filtered observables (deviations from steady state in per cent)

- $y_{obs}$ ... observed real output
- $r_{obs}$ ... observed interest rate
- $\pi_{obs}$ ... observed CPI inflation
- $\text{inv}_{obs}$ ... observed real investment
- $y^*_{obs}$ ... observed foreign real output
- $r^*_{obs}$ ... observed foreign interest rate
- $\pi^*_{obs}$ ... observed foreign CPI inflation
- $r_{rer obs}$ ... observed real exchange rate
## Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ Discount factor</td>
<td>0.995</td>
</tr>
<tr>
<td>$\alpha$ Capital share in production</td>
<td>0.350</td>
</tr>
<tr>
<td>$\delta$ Capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\mu$ Steady-state domestic mark-up</td>
<td>1.200</td>
</tr>
<tr>
<td>$\Omega$ Household’s share in labour supply</td>
<td>0.990</td>
</tr>
</tbody>
</table>
## Estimation results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Prior Mean</th>
<th>Std</th>
<th>CZ Posterior Mean</th>
<th>Std</th>
<th>EA Posterior Mean</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma ) Habit persistence</td>
<td>B</td>
<td>0.60</td>
<td>0.05</td>
<td>0.60</td>
<td>0.05</td>
<td>0.68</td>
<td>0.06</td>
</tr>
<tr>
<td>( \psi ) Inv. elast. of lab. supply</td>
<td>G</td>
<td>2.00</td>
<td>0.50</td>
<td>1.25</td>
<td>0.35</td>
<td>0.88</td>
<td>0.26</td>
</tr>
<tr>
<td>( \psi^B ) Debt-elastic risk premium</td>
<td>G</td>
<td>0.05</td>
<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>( \eta ) Home/foreign elast. subst.</td>
<td>G</td>
<td>0.65</td>
<td>0.10</td>
<td>0.52</td>
<td>0.08</td>
<td>0.43</td>
<td>0.02</td>
</tr>
<tr>
<td>( \kappa ) Price indexation</td>
<td>B</td>
<td>0.50</td>
<td>0.10</td>
<td>0.49</td>
<td>0.09</td>
<td>0.44</td>
<td>0.09</td>
</tr>
<tr>
<td>( \gamma ) Pref. bias to foreign goods</td>
<td>B</td>
<td>0.40</td>
<td>0.15</td>
<td>0.48</td>
<td>0.07</td>
<td>0.27</td>
<td>0.04</td>
</tr>
<tr>
<td>( \theta_H ) Home goods Calvo</td>
<td>B</td>
<td>0.70</td>
<td>0.10</td>
<td>0.82</td>
<td>0.03</td>
<td>0.80</td>
<td>0.03</td>
</tr>
<tr>
<td>( \theta_F ) Foreign goods Calvo</td>
<td>B</td>
<td>0.70</td>
<td>0.10</td>
<td>0.84</td>
<td>0.02</td>
<td>0.81</td>
<td>0.03</td>
</tr>
<tr>
<td>( \psi^I ) Capital adjustment costs</td>
<td>G</td>
<td>8.00</td>
<td>3.00</td>
<td>11.5</td>
<td>2.92</td>
<td>15.5</td>
<td>3.35</td>
</tr>
<tr>
<td>( \Gamma ) Leverage ratio ss ratio</td>
<td>G</td>
<td>2.00</td>
<td>0.50</td>
<td>1.41</td>
<td>0.24</td>
<td>1.16</td>
<td>0.21</td>
</tr>
<tr>
<td>( \varsigma ) Bankruptcy rate</td>
<td>B</td>
<td>0.025</td>
<td>0.015</td>
<td>0.05</td>
<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>( \chi ) Financial accelerator</td>
<td>G</td>
<td>0.05</td>
<td>0.015</td>
<td>0.04</td>
<td>0.01</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>Taylor rule</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho ) Interest rate smoothing</td>
<td>B</td>
<td>0.70</td>
<td>0.10</td>
<td>0.86</td>
<td>0.02</td>
<td>0.74</td>
<td>0.04</td>
</tr>
<tr>
<td>( \beta_\pi ) Inflation weight</td>
<td>G</td>
<td>1.50</td>
<td>0.20</td>
<td>1.75</td>
<td>0.23</td>
<td>1.76</td>
<td>0.23</td>
</tr>
<tr>
<td>( \Theta_y ) Output gap weight</td>
<td>G</td>
<td>0.50</td>
<td>0.20</td>
<td>0.16</td>
<td>0.05</td>
<td>0.22</td>
<td>0.06</td>
</tr>
</tbody>
</table>
## Estimation results, shock parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Prior Mean</th>
<th>Std</th>
<th>Posterior Mean CZ</th>
<th>EA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation coefficients</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_Y$ Domestic productivity</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.58</td>
<td>0.47</td>
</tr>
<tr>
<td>$\rho_{UIP}$ Uncovered interest parity</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.63</td>
<td>0.79</td>
</tr>
<tr>
<td>$\rho_{LOP}$ Law of one price</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.93</td>
<td>0.84</td>
</tr>
<tr>
<td>$\rho_{NW}$ Entrepreneurial net worth</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.44</td>
<td>0.40</td>
</tr>
<tr>
<td>Standard deviations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_Y$ Domestic productivity</td>
<td>IG</td>
<td>1.00</td>
<td>$\infty$</td>
<td>1.09</td>
<td>0.38</td>
</tr>
<tr>
<td>$\sigma_{UIP}$ Uncovered interest parity</td>
<td>IG</td>
<td>0.50</td>
<td>$\infty$</td>
<td>0.27</td>
<td>0.24</td>
</tr>
<tr>
<td>$\sigma_{LOP}$ Law of one price</td>
<td>IG</td>
<td>0.50</td>
<td>$\infty$</td>
<td>3.22</td>
<td>5.05</td>
</tr>
<tr>
<td>$\sigma_{NW}$ Entrepreneurial net worth</td>
<td>IG</td>
<td>1.00</td>
<td>$\infty$</td>
<td>1.84</td>
<td>1.48</td>
</tr>
<tr>
<td>$\sigma_{MP}$ Monetary policy</td>
<td>IG</td>
<td>0.50</td>
<td>$\infty$</td>
<td>0.08</td>
<td>0.10</td>
</tr>
<tr>
<td>$\sigma_y^\ast$ Foreign output</td>
<td>IG</td>
<td>1.00</td>
<td>$\infty$</td>
<td>0.52</td>
<td>0.54</td>
</tr>
<tr>
<td>$\sigma_{\pi}^\ast$ Foreign inflation</td>
<td>IG</td>
<td>0.50</td>
<td>$\infty$</td>
<td>0.14</td>
<td>0.30</td>
</tr>
<tr>
<td>$\sigma_{r}^\ast$ Foreign interest rate</td>
<td>IG</td>
<td>0.50</td>
<td>$\infty$</td>
<td>0.08</td>
<td>0.10</td>
</tr>
</tbody>
</table>
Measurement errors (deviations from steady state in per cent)

- Real output error: $y_{obs} - y$
- Foreign real output error: $y^*_{obs} - y^*$
- Interest rate error: $r_{obs} - r$
- Foreign interest rate error: $r^*_{obs} - r^*$
- CPI inflation error: $\pi_{obs} - \pi$
- Foreign CPI inflation error: $\pi^*_{obs} - \pi^*$
- Real investment error: $inv_{obs} - inv$
- Real exchange rate error: $rer_{obs} - rer$
Filtered shock innovations (deviations from steady state in per cent)

\( \varepsilon_y \) ... productivity innovations
\( \varepsilon_{MP} \) ... monetary policy innovations
\( \varepsilon_{UIP} \) ... UIP innovations
\( \varepsilon_{LOP} \) ... LOP innovations
\( \varepsilon_{NW} \) ... survival rate innovations
\( \varepsilon_{y^*} \) ... foreign output innovations
\( \varepsilon_{r^*} \) ... foreign interest rate innovations
\( \varepsilon_{\pi^*} \) ... foreign inflation innovations
Selected filtered variables (deviations from steady state in per cent)
Time-varying parameters (deviations from initial value in per cent)

- Financial accelerator ($\chi$)
- Bankruptcy rate ($\zeta$)
- Capital adjustment costs ($\Psi_I$)
- Leverage rate ($\Gamma$)
- Foreign goods preference bias ($\gamma$)
- Domestic Calvo parameter ($\theta_H$)
- Taylor rule, inflation ($\beta_\pi$)
- Taylor rule, output gap ($\Theta_y$)
- Taylor rule, smoothing ($\rho$)
### Correlation of time-varying parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi$</td>
<td>0.66</td>
</tr>
<tr>
<td>$\varsigma$</td>
<td>-0.13</td>
</tr>
<tr>
<td>$\Psi_I$</td>
<td>0.71</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>0.64</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.19</td>
</tr>
<tr>
<td>$\theta_H$</td>
<td>0.34</td>
</tr>
<tr>
<td>$\beta_\pi$</td>
<td>-0.20</td>
</tr>
<tr>
<td>$\Theta_y$</td>
<td>0.25</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-0.18</td>
</tr>
</tbody>
</table>
Results of the estimation suggest that some structural changes occurred during recent financial and economic crisis.

The structural changes were probably temporary as the parameters tend to return to their initial values.

Some parameters showed only negligible deviations from their initial values (elasticity of intertemporal substitution, risk premium elasticity, inflation indexation).

Some parameters of the financial sector, openness parameter, Calvo parameters and interest rate smoothing parameter changed markedly during the recent economic crisis.
Overall, the estimated trajectories show many similarities between the development in the Czech economy and in the euro area with some differences in the magnitude of the deviations and timing.

The differences can be attributed to earlier onset and more dramatic course of the financial crisis in the euro area than in relatively sheltered Czech economy.

The trajectories of the Taylor rule parameters also show interesting differences in the behaviour of the ECB and CNB.
References


Thank you for your attention!

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