THE SIMPLIFIED DYNAMIC MODEL OF A ROBOT MANIPULATOR

J. Kardoš

Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava, Slovak Republic

Abstract

The analytical model of a robot dynamics represents an important tool for both the analysis and the synthesis of robot control algorithms. Based on the Euler-Lagrange formalism, the contribution presents a MATLAB-SIMULINK dynamic model of the 3-DOF anthropomorphic robot manipulator with revolute joints. A significant simplification of the model has been accomplished via the concentration of the mass of each link into its centre of gravity.

1 Problem Description

Robot manipulators represent a class of complex dynamic systems with extremely variable inner parameters (e.g. the radical variation of the inertia matrix elements within the manipulation process) as well as the intensive contact with the environment. An accurate control of such a complex system deals with the problem of parametric uncertainty, the natural and technological limitation of the signals and variables, and the mutual interaction among the kinematic chain components. The direct implementation of control algorithms in the control system of a real manipulator is impossible without a preceding thorough verification of the robot behavior using the correct dynamic model.

Generally, for any mechatronic system, two different approaches to the dynamic model creation are possible – the Newton-Euler and the Euler-Lagrange formalisms [1]. Specifically in the field of robot manipulators, the first one starts with the description of each robot link motion using the Newton's laws. Then, using the forward-backward recursion, the coupling among the links is established. The second one treats the robot as a whole, taking its kinetic and potential energy into account. Nevertheless, the results of both methods are equivalent. In the author's view, the Euler-Lagrange formulation gives a more compact and direct methodology of the dynamics analysis.

The kinetic energy of any moving material body is given as the quadratic function of both the velocity (the change of position, i.e. translation) and the rotational velocity (the change of orientation, i.e. rotation) [2]. One of the steps in a common simplified approach to the physical object description is the concentration of its mass to the centre of gravity. Thus, the object is represented by a mass point which motion is fully described by the velocity vector tangential to the trajectory of the motion. This simplification eliminates the change of orientation from the solution.

The tangential velocity vector of any mass point can be derived from its position vector with respect to a common coordinate system. To get the position vector of any point of the manipulator kinematic chain, a coordinate frame of each joint should be established. Subsequently, the homogeneous transformation among the frames gives the position vector as a function of the joint variables.

2 Kinematics of the 3-DOF Robot Manipulator

The kinematic chain of the 3-DOF robot manipulator can be seen in Figure 1. Let its rotational basis *AB* (length l_1) with the revolute joint *A* be located in the vertical z_0-axis of the common coordinate system ($x_0y_0z_0$). The axis of the second revolute joint *B* is perpendicular to the z_0-axis. The mass m_2 of the second link *BC* (length l_2) is concentrated in the centre of gravity M_2. The axis of the third revolute joint *C* is parallel to the axis of the joint *B*. Similarly to the second link, the mass m_3 of the third link *CD* (length l_3) is concentrated in the centre of gravity M_3. The positive sense of the joint variables q_1 (joint *A*), q_2 (joint *B*) and q_3 (joint *C*) is given by the right hand rule.

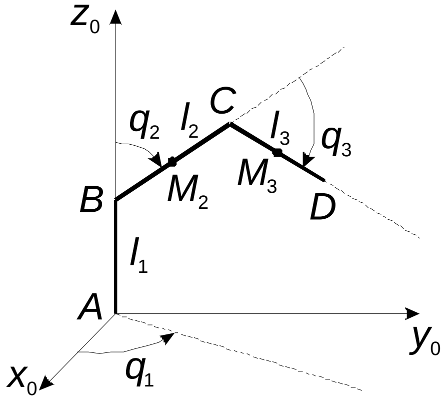

Figure 1: Kinematic chain of the 3-DOF robot manipulator

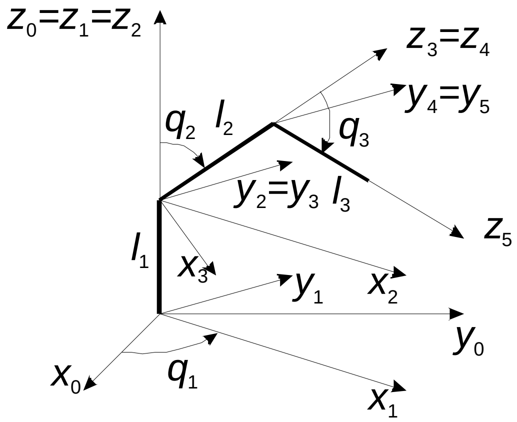

Figure 2: System of the joint coordinate frames

In Figure 2, two individual coordinate frames are attached to any single link – the first frame is defined for the joint rotation and the second one for the constructional displacement. Thus, the homogeneous transformation matrix $\mathbf{T}_{i-1,i}$ between the neighboring i-1 and i frames represents the elementary rotational matrix $\mathbf{R}_{axis}(q_i)$ or the translational one $\mathbf{T}_{axis}(l_i)$. The complete set of the transformation matrices related to Figure 2 is as follows

$$\mathbf{T}_{01}(q_1) = \mathbf{R}_z(q_1) = \begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 & 0 \\ \sin(q_1) & \cos(q_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{T}_{12}(l_1) = \mathbf{T}_z(l_1) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{T}_{23}(q_2) = \mathbf{R}_y(q_2) = \begin{pmatrix} \cos(q_1) & 0 & \sin(q_1) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(q_1) & 0 & \cos(q_1) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{T}_{34}(l_2) = \mathbf{T}_z(l_2) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

$$\mathbf{T}_{45}(q_3) = \mathbf{R}_y(q_3) = \begin{pmatrix} \cos(q_3) & 0 & \sin(q_3) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(q_3) & 0 & \cos(q_3) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The position vectors of the mass points M_2 and M_3 in the appropriate joint frames follow from Figure 1 and Figure 2

$$\mathbf{p}_{M_2 3} = \begin{pmatrix} 0 \\ 0 \\ \frac{l_2}{2} \\ 1 \end{pmatrix} \qquad \mathbf{p}_{M_3 5} = \begin{pmatrix} 0 \\ 0 \\ \frac{l_3}{2} \\ 1 \end{pmatrix} \tag{2}$$

The resultant position vectors of the mass points M_2 and M_3 in the common coordinate system according to (1) and (2) are

$$\mathbf{p}_{M_2 0} = \begin{pmatrix} \frac{l_2}{2}\sin(q_2)\cos(q_1) \\ \frac{l_2}{2}\sin(q_2)\sin(q_1) \\ l_1 + \frac{l_2}{2}\cos(q_2) \\ 1 \end{pmatrix} \qquad \mathbf{p}_{M_3 0} = \begin{pmatrix} \left(l_2\sin(q_2) + \frac{l_3}{2}\sin(q_2+q_3)\right)\cos(q_1) \\ \left(l_2\sin(q_2) + \frac{l_3}{2}\sin(q_2+q_3)\right)\sin(q_1) \\ l_1 + l_2\cos(q_2) + \frac{l_3}{2}\cos(q_2+q_3) \\ 1 \end{pmatrix} \tag{3}$$

3 Dynamics of the 3-DOF Robot Manipulator

Dynamic model of the above specified type of manipulator has been derived using the Euler-Lagrange formulation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \tag{4}$$

where $L = K - P$ is the Lagrangian (K and P stand for the kinetic and the potential energy of the whole dynamic system respectively), q_i denotes the generalized coordinate of the i^{th} joint (i.e. the joint variable), Q_i is for the generalized forces in the i^{th} joint (for example driving torque τ_i). Note that the source of the unavoidable gravitational forces is the potential energy at the left-hand side of (4). Furthermore, to find a realistic model of the robot manipulator, the right-hand side of equation (4) should contain also the term corresponding with the dissipative forces, for example the viscous friction $B_i\dot{q}_i$.

Kinetic energy of the moving mass point M_i is a function of the squared module of the tangential velocity $\mathbf{v}_i$ which with respect to (3) can be obtained for the mass point M_2 in the form

$$abs^2(\mathbf{v}_2) = \left(\frac{l_2}{2}\sin(q_2)\frac{dq_1}{dt}\right)^2 + \left(\frac{l_2}{2}\frac{dq_2}{dt}\right)^2 \tag{5}$$

and for the mass point M_3 in the form

$$abs^2(\mathbf{v}_2) = \left(l_2\sin(q_2) + \frac{l_3}{2}\sin(q_2+q_3)\right)^2\left(\frac{dq_1}{dt}\right)^2 + \left(l_2^2 + \left(\frac{l_3}{2}\right)^2 + l_2 l_3\cos(q_3)\right)\left(\frac{dq_2}{dt}\right)^2 + \\ + \left(\frac{l_3}{2}\right)^2\left(\frac{dq_3}{dt}\right)^2 + \left(\frac{1}{2}l_3^2 + l_2 l_3 coc(q_3)\right)\frac{dq_2}{dt}\frac{dq_3}{dt} \tag{6}$$

It is evident, that the solution of expression (4) for the 3-DOF manipulator represents the system of 3 second-order differential equations with variable parameters. This one can be written in the form of the 2nd order matrix differential equation

$$\mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} = \tau - \mathbf{B}\dot{\mathbf{q}} - \mathbf{c}(\mathbf{q},\dot{\mathbf{q}}) - \mathbf{g}(\mathbf{q}) \tag{7}$$

with the vector of generalized coordinates $\mathbf{q} \in R^3$, the vector of generalized driving forces (torques) $\tau \in R^3$, the inertia matrix $\mathbf{J} \in R^{3\times3}$, the diagonal matrix of damping (viscous friction) coefficients $\mathbf{B} \in R^{3\times3}$, the vector of Coriolis and centrifugal forces $\mathbf{c} \in R^3$, and the vector of gravitational forces $\mathbf{g} \in R^3$.

Due to specific orientation of the revolute joint axes, some elements of equation (7) are quite simple. Only the diagonal elements j_{ii} ($i = 1, 2, 3$) of the inertia matrix $\mathbf{J}$ and the elements j_{23} and j_{32} are the non-zero elements. Moreover, the elements of matrix $\mathbf{J}$ are independent of the variable q_1 and the following formulas are valid: $j_{11} = f(q_2, q_3)$, $j_{22}, j_{23}, j_{32} = f(q_3)$, $j_{33} = const.$ Because of the parallel orientation of the second and the third revolute joints, there is a significant inertial interaction between the corresponding links but this interaction is mutually equivalent, i.e. $j_{23} = j_{32}$.

As for gravitational forces, the first element of vector $\mathbf{g}$ is zero, i.e. $g_1 = 0$ due to vertical orientation of the first joint axis. The Coriolis and centrifugal term $\mathbf{c}$ is independent of the variable q_1 but the influence of the joint velocities is complete.

The final structure of the MATLAB-SIMULINK model of the 3-DOF robot manipulator dynamics can be seen in Figure 3. Figure shows the characteristic vertical structure of three links. Each link is represented by the individual group of objects with the input driving torque (*tau*) on the left-hand side and the set of output signals (the angular position *q*, the angular velocity *q’* and the angular acceleration *q’’*) on the right-hand side. The upper group (index 1) stands for the first link and the lower group (index 3) is for the third link. The interaction between any pair of links is given by the output signal coupling.

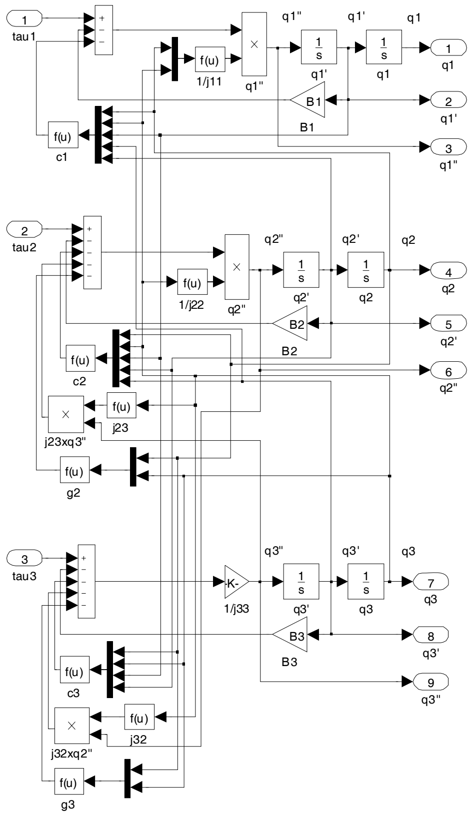

Figure 3: MATLAB-SIMULINK structure of the 3-DOF dynamic model

4 Experimental Results

Dynamic model of the 3-DOF robot manipulator has been utilized in the synthesis process of various robust motion control algorithms. In Figure 4, there is an example of a perfect command tracking ability using the sliding mode control [3] within the drop down phase of the manipulator positioning. The upper part of figure shows the command (desired value q_d) and the controlled variable (angle position *q*) plots in time domain. The left-hand side graph corresponds to the first link motion, the middle graph to the second link motion, and the right-hand side graph to the third one. The lower part of Figure 4 shows the corresponding driving torque versus time diagrams.

At the beginning of the control process, the starting position of all manipulator links is vertical ($q = 0$ and $d_d = 0$), i.e. there is no gravitation influence in any joint. This implies the zero value of the driving torques ($\tau = 0$). In contrast, the final position of the neighboring links should be perpendicular to each other ($q_d = \pi/2$), i.e. there is a maximal gravitation influence in the second joint and no influence in the third one. This corresponds with a non-zero final value of the driving torque in the second DOF and a zero driving torque in the third DOF (cf. Figure 4).

It is evident, that the interval of the maximal positive value of a driving torque corresponds with the period of the link acceleration. Similarly, the maximal negative impulse of the torque value appears at the beginning of the breaking period. Nevertheless, the considered direction of the manipulator motion shows a strong additional influence of the gravitational forces (the same direction of the gravitation and the motion) on the second and the third link behavior. Thus, the compensation of gravitational forces implies the mostly negative values of the driving torques for the period of the positioning.

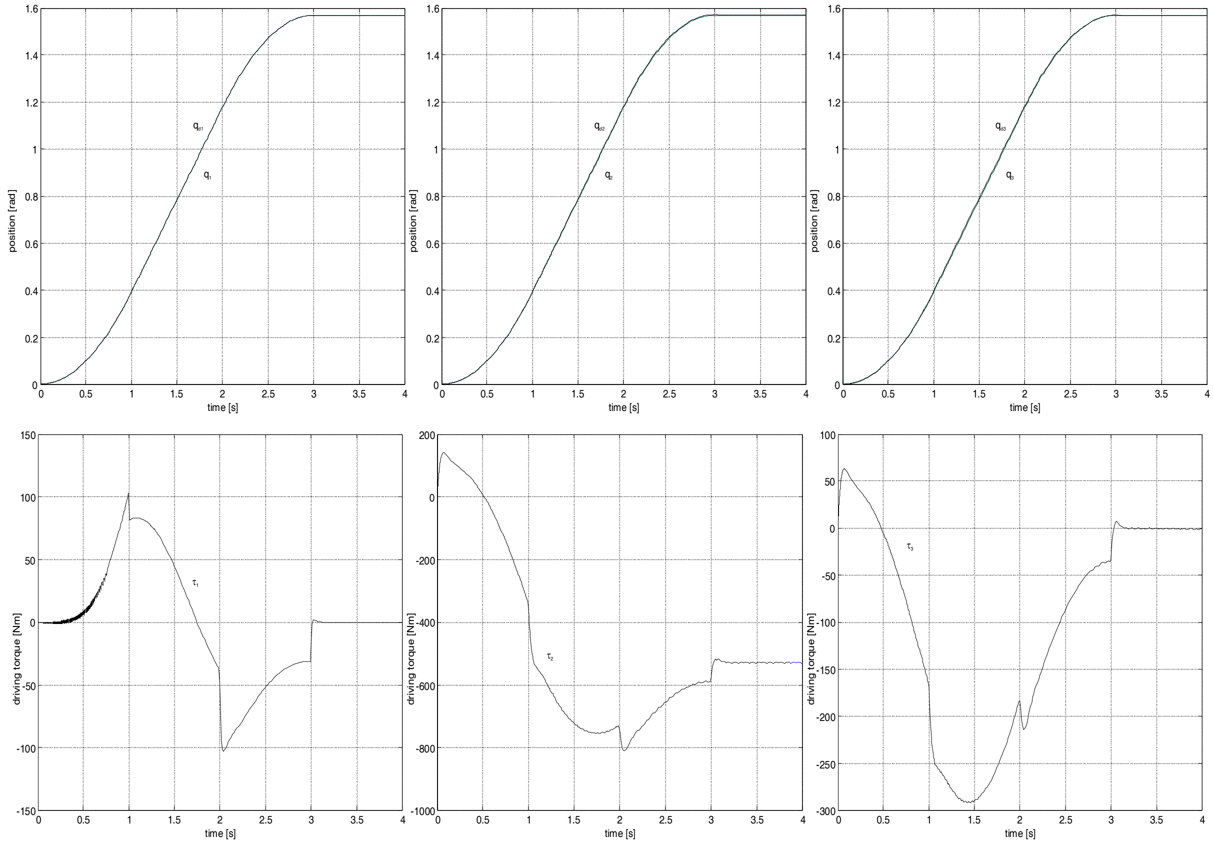

Figure 4: Command tracking in 3-DOF robot manipulator (drop down phase)

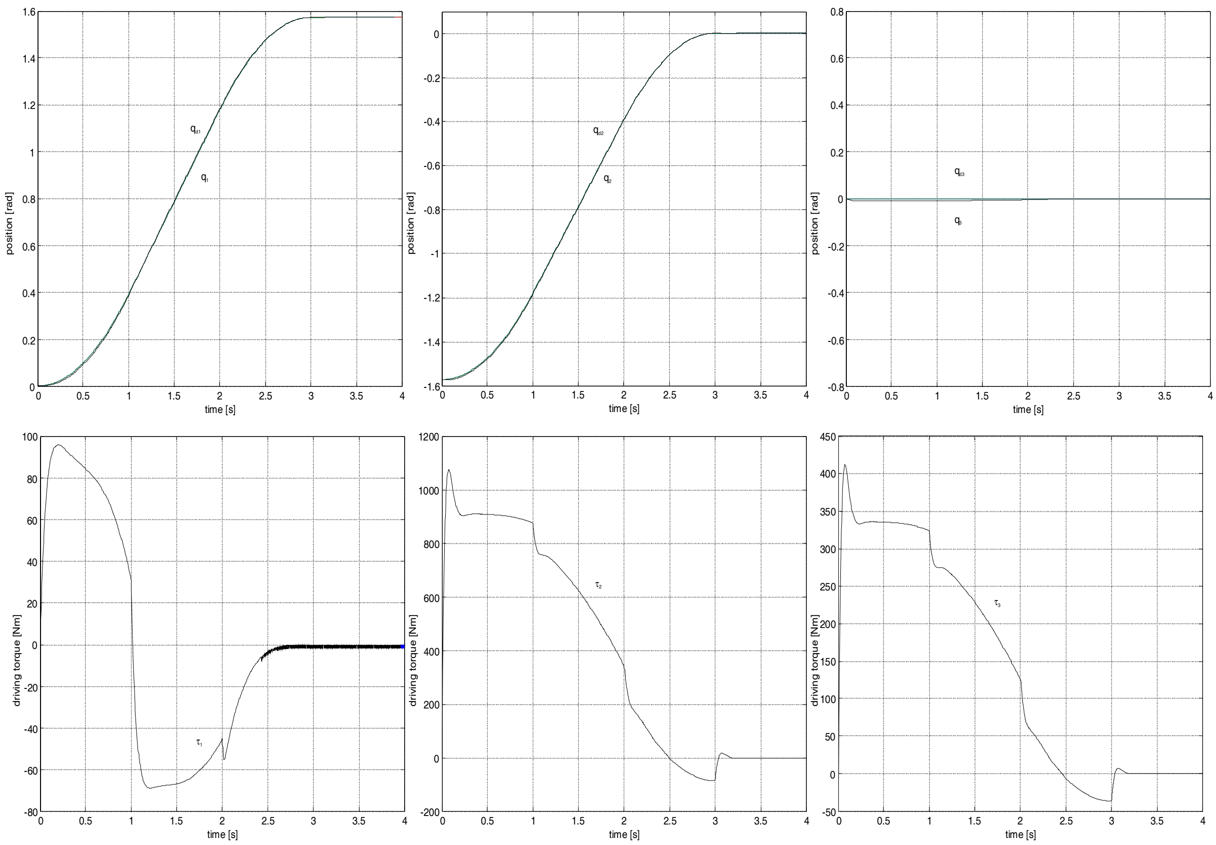

Figure 5: Command tracking in 3-DOF robot manipulator (pick up phase)

Figure 5 illustrates the behavior of the 3-DOF manipulator dynamics during the pick up phase of the positioning. The initial position of the second and the third manipulator link is horizontal ($q_1 = q_{d1} = -\pi/2$ and $q_2 = q_{d2} = 0$). In this case, to accelerate the manipulator from the starting position and to overcome the gravitational forces influence, the driving torque in the second joint should reach the maximal value (cf. the starting positive impulse of the corresponding torque in Figure 5). For the reason that the constant (zero) desired value of the third link has been chosen, i.e. the mutual position of the second and the third link remains the same during the whole transient process, the driving torque in the third joint should have the behavior similar to the torque in the preceding joint (cf. Figure 5). This is the direct evidence of the intensive inertial influence between the second and the third link. Within the breaking phase, gravitational forces help the driving motors in the second and the third DOF to slow-down the motion. Thus, the amplitudes of the driving torques have the small negative values. The high starting value of the driving torque in the first joint (Figure 5) is the consequence of the maximal inertia torque due to the horizontal position of the second and the third link, which is the opposite case of the situation in Figure 4. At the end of the positioning, there is no influence of the gravitation forces in any joint. This implies the zero values of driving torques in steady state.

The high-frequency oscillation (chattering) of the driving torque in the first DOF (cf. the starting phase of the motion in Figure 4 and the steady state in Figure 5) is the consequence of the applied control algorithm – the sliding mode control. The elimination of chattering is beyond the scope of the presented contribution. However, this oscillation has no influence on the tracking quality.

5 Conclusions

The contribution presents the main features of a 3-DOF manipulator dynamic model synthesis using the Euler-Lagrange formalism and the homogeneous transformation matrices. The MATLAB-SIMULINK model structure is provided and the model is demonstrated by means of a robust command following control algorithm. Such a type of control with an unambiguous correlation between the desired value and the driving torque enables an exact interpretation of the experimental results with respect to the verification of the synthesized dynamic model. The torque versus time diagram shows an acceptable behavior of the presented model.

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Ján Kardoš
Institute of Control and Industrial Informatics, Faculty of Electrical Engineering and Information Technology, STU in Bratislava, Ilkovičova 3, 81219 Bratislava, Slovak Republic
jan.kardos@stuba.sk