JUSTIFICATION OF INPUT AND OUTPUT CONSTRAINTS INCORPORATION INTO PREDICTIVE CONTROL DESIGN

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Abstract

Model predictive control has proved its effectiveness in many industrial applications. An important feature of this control design approach is that the process variables constraints can be systematically taken into account in the design procedure. The aim of this paper is to analyze how the incorporation of the input/output limits influences the resulting control system performances and the control design complexity.

1 Introduction

Research in the area of control theory has brought a number of advanced control techniques that can be used to improve the control performances. Model predictive control (MPC) is well known control design approach, which makes explicit use of process model to predict the future process behavior and based on this prediction future control sequence minimizing the objective function is calculated [1]. The MPC popularity in both the research and industrial community is mainly due to its relatively simple time-domain formulation and good performance. As in practice all processes are subject to the input and output constraints, another important feature is that these constraints can be systematically incorporated into the design procedure.

In predictive control design there are several tuning parameters that influence the resulting closed loop performance and stability properties. In case of constrained input/output signals these parameters can be chosen so as the constraints are not exceeded which on the other hand can deteriorate the performances inside the operating range. If the constraints are taken into account in the control design procedure, the resulting algorithm is more complex and time consuming. That is why the paper deals with the justification of the constraints incorporation into the predictive control design procedure.

In the paper the bounds in the amplitude and in the slew rate of the control signal and limits in the output have been considered. Besides the simulations also the real time experiments have been performed. To their realization the Matlab environment together with the programmable logic controller Simatic S7-200 have been used.

The paper is organized as follows. First, standard GPC algorithm is briefly presented. Then its modification taking into account the process variables constraints is described. The properties of both control algorithms are analyzed using the simulation examples and the real-time control of a laboratory plant.

2 Generalized predictive control

Generalized predictive control (GPC) [2] is one of the most popular predictive algorithms based on the parametric input/output process model. It is applicable to the wide class of processes including the instable or inversely instable processes or processes with unknown or variable time delay. The GPC implementation is relatively simple and thanks to several control design parameters the control law can be tuned to specific applications.

2.1 Plant model

Consider that the process behavior can be described by the following CARIMA model

$$A(z^{-1})y(t) = B(z^{-1})u(t-d-1) + v(t) + w(t) \quad (1)$$

$$D(z^{-1})v(t) = C(z^{-1})\xi(t) \quad (2)$$

where *u(t)* is the control signal, *y(t)* is the measured plant output, *d* denotes the minimum plant model time-delay in sampling periods, *v(t)* represents the external disturbances and *ξ(t)* is the random variable with zero mean value and finite variance. Of practical importance $D\left(z^{-1}\right)=1-z^{-1}$ allows to incorporate an integral action into the design. For simplicity in the following the $C(z^{-1})$ polynomial is chosen to be 1.

2.2 j-step ahead prediction

The GPC control design procedure requires the calculation of the sequence of *j*-step ahead future values of the plant output (predictions) which are composed of the following two parts:

- free response (depends on data available up to the time *t*, namely the past control signal and output values),
- forced response (depends on the future control signal values that can be obtained by the minimization of the GPC control objective),

while the elements that depend on the random disturbance *ξ(t)* are unpredictable and are omitted. The calculation of the *j*-step ahead output predictions necessitates the iterative solutions of the following Diophantine equations for $j = sh,\dots,ph$

$$1 = A(z^{-1})D(z^{-1})E_j(z^{-1}) + z^{-j}F_j(z^{-1}) \tag{3}$$

$$E_j(z^{-1})B(z^{-1}) = G_{j-d}(z^{-1}) + z^{-j+d}H_{j-d}(z^{-1}) \tag{4}$$

2.3 Standard control design

The control objective consists of minimizing in a receding horizon sense the following cost function

$$J(t,ph,ch,sh,\rho) = E\left\{\sum_{j=sh}^{ph}\left(\hat{y}(t+j/t)-y^*(t+j)\right)^2 + \rho\left(D\left(z^{-1}\right)u(t+j-sh)\right)^2\right\} \tag{5}$$

subject to

$$D\left(z^{-1}\right)u(t+i) = 0 \qquad for \qquad ch \le i \le ph \tag{6}$$

sh, *ph* and *ch* are positive scalars defining the starting horizon, prediction horizon and control horizon, ρ is a nonnegative control weighting scalar. $\hat{y}(t+j/t)$ denotes the j-step ahead prediction of *y(t)* based on the data available up to time *t* and $y^*(t+j)$ is the future reference value which is supposed to be known.

The GPC control design then consists in performing the following three steps:

- Compute the j-step ahead output predictions $\hat{y}(t+j/t)$ for $j = sh,\dots,ph$.
- Minimize with respect to the future control signal sequence the objective function (5) – (6).
- Only the first component out of this sequence is used for control

$$D\left(z^{-1}\right)u(t) = -\sum_{j=sh}^{ph}\gamma_j\left(y_0(t+j/t)-y^*(t+j)\right) \tag{7}$$

  where the coefficients γ_j depend on the solutions of Diophantine equations (3) – (4).

The whole process will be repeated in the next sampling period.

The control law (7) may be implemented using the standard pole-placement control structure (depicted in Fig. 1)

$$S\left(z^{-1}\right)D\left(z^{-1}\right)u(t) + R\left(z^{-1}\right)y(t) = T\left(z^{-1}\right)y^*(t) \tag{8}$$

The polynomials $R(z^{-1})$, $S(z^{-1})$ and $T(z^{-1})$ need to be calculated only once before the control law implementation and for the control signal calculation at the time *t* only the output and control signals available up to the time *t* as well as the reference value $y^*(t)$ are necessary. The orders of $R(z^{-1})$, $S(z^{-1})$ and $T(z^{-1})$ polynomials are given by orders of the plant model numerator and denominator and their coefficients depend on the plant model parameters as well as on the choice of the tuning parameters *sh*, *ph*, *ch* and ρ.

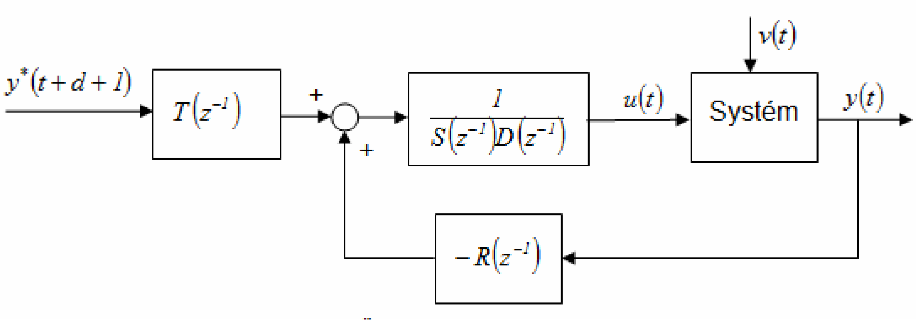

Figure 1: Control scheme

2.4 Control design taking into account the input/output constraints

In practice all process variables (process output, control signal) are subject to constraints. Constructive, safety or environmental reasons and sensor scopes can cause the limits on the process variables. If these constraints are not taken into account in the control design but they are active during the control implementation, it can lead to deterioration of the control performances or to instability.

For this reason it is reasonable to introduce the constraints into the function to be minimized. Normally, bounds in the amplitude and in the slew rate of the control signal and limits in the output are considered

$$u_{min} \le u(t+j/t) \le u_{max} \quad j = 0,1,\ldots,ch-1 \tag{9}$$

$$|\Delta u(t+j/t)| \le \Delta u_{max} \quad j = 0,1,\ldots,ch-1 \tag{10}$$

$$y_{min} \le \hat{y}(t+j/t) \le y_{max} \quad j = 0,1,\ldots,ph \tag{11}$$

By adding the constraints (9) – (11) to the objective function (5) – (6) the minimization becomes more complex, so the solution can not be obtained explicitly as in the unconstrained case and the control law can not be expressed in the simple RST form (8). The quadratic programming (QP) methods have to be used to solve this optimization problem. The numerical optimization is carried out each sampling instant and the obtained optimal value of *u(t)* is sent to the process. In the following this control design will be denoted as GPC QP.

2.5 Implementation

Implementation of standard GPC control law (8) not considering the input/output constraints in the design procedure is very simple. As it has already been mentioned, the calculation of the polynomials $R(z^{-1})$, $S(z^{-1})$ and $T(z^{-1})$ necessitates the recursive solution of Diophantine equations (3) – (4) which can represent a great amount of calculations especially in case of large prediction horizons, but these calculations are performed offline before the control application. The control signal in each sampling instant is computed as a linear combination of past output and control signal values, current output value and the reference value.

GPC QP implementation needs the calculation of the process free response which has been realized using the S-function. The computational effort is much higher comparing to the implementation of the standard constraint-free control law because the solution of the QP optimization problem has to be found each sampling instant. To this end, the Matlab function *quadprog* has been used.

3 Case study

In order to evaluate the influence of the input/output constraints on the control performance the simulation of simple cylindrical tank depicted in Fig. 2 has been performed. The tank parameters are given in Table 1. The tank is a nonlinear system whose time constant and gain vary considerably throughout the operating range.

The controlled variable is the liquid height h and the control variable is the inlet flow rate Q_1 with the operating range from 0 to $0.04\ m^3/s$. The outlet flow rate Q_2 depends on the liquid height according to the Torricelli's law. The control objective is to follow the changes of liquid height reference value as well as to reduce the effect of disturbance represented by the additional inlet flow rate Q_3 subject to the input/output constraints as well as changing plant dynamics caused by the operating point changes.

TABLE 1: TANK PARAMETERS

$S = 1\ m^2$	$S_0 = 100\ cm^2$
$\mu = 0{,}62$	$\rho = 1000\ \ kg\,m^{-3}$
$g = 9{,}81\ m\,s^{-2}$	$h_{\max} = 2\ m$

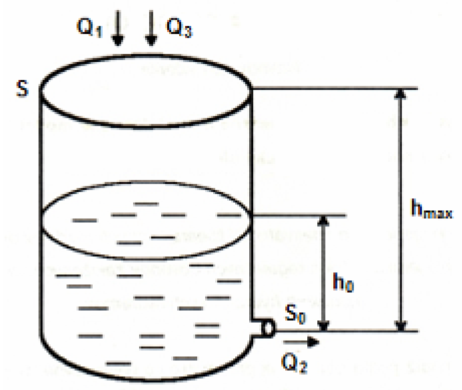

Figure 2: Cylindrical tank

For the control design purposes the following model of the liquid height dynamics has been identified around an operating point given by $h_0 = 0.5\ m$ with the sampling time $T_s = 10\,s$

$$G_s\left(z^{-1}\right) = \frac{8.772z^{-1}}{-0.8297z^{-1} + 1} \tag{12}$$

Based on this model, three simulations have been performed.

First, the standard GPC controller has been designed. If the input and output limits are reached during the regulation, the control performances are deteriorated. To avoid this undesirable effect, the choice of the control tuning parameters (i.e. control weighting and horizons) can influence the control system response so as the constraints are not reached. However, this solution leads also to the control performance deterioration. Finally, it is shown that using the GPC QP design it is possible to obtain good control performances even under active input/output constraints.

3.1 First simulation – standard GPC

In this simulation the control tuning parameters have been set to the following values

- $ph = 3,$
- $ch = 1,$
- $ro = 0.$

Setting the prediction horizon to small value results in fast controller response manifested by large liquid height overshoot. If the calculated control signal – the inlet flow rate Q_1 – exceeds its limits, it is truncated to the corresponding limiting value, which, as a consequence, leads to the worse control performances. If the control law includes an integrator, the calculated control signal will continuously rise although its real implemented value is constant and equal to the limiting value. This behavior is known as the wind-up effect.

The influence of the control signal magnitude constraints on the control performances can be seen in Fig. 3. The additional inlet flow rate $Q_3 = 0{,}017\ m^3\ s^{-1}$ acts as a disturbance from the time 850 s. The blue line shows the simulation results of the unconstrained case (i.e. constraints are not active), while the simulation results where the control signal has been truncated to its limiting values are represented by red line.

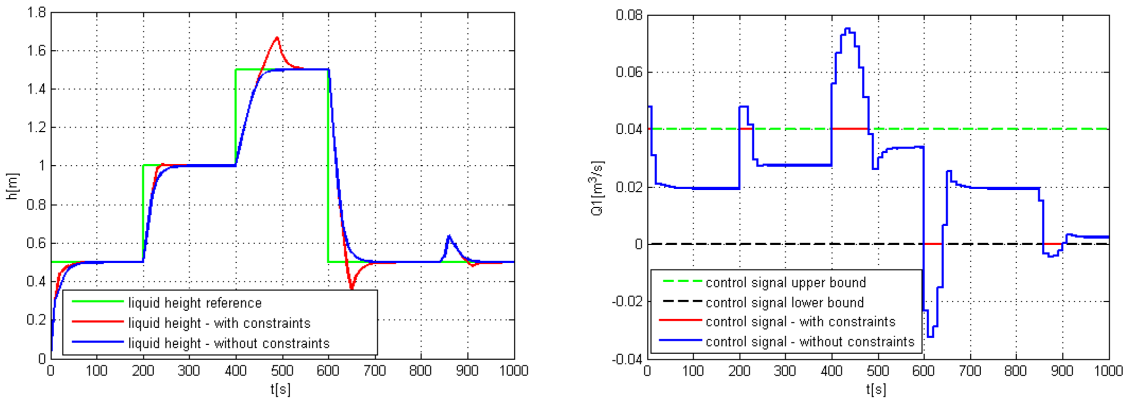

Figure 3: First simulation results

3.2 Second simulation – standard GPC with modified parameters

In this simulation the control tuning parameters have been chosen so as the calculated control signal do not exceed given constraints during the standard operation

- $ph = 15,$
- $ch = 1,$
- $ro = 200.$

The simulation results (blue line) compared to the first simulations results (red line) are in Fig. 4. It can be seen that in the second simulation the wind-up effect has been eliminated, but the settling time has been much longer (for the liquid height reference value step change from 0.5 m to 1 m it has been about 90 s, while in the first simulation approximately 27 s).

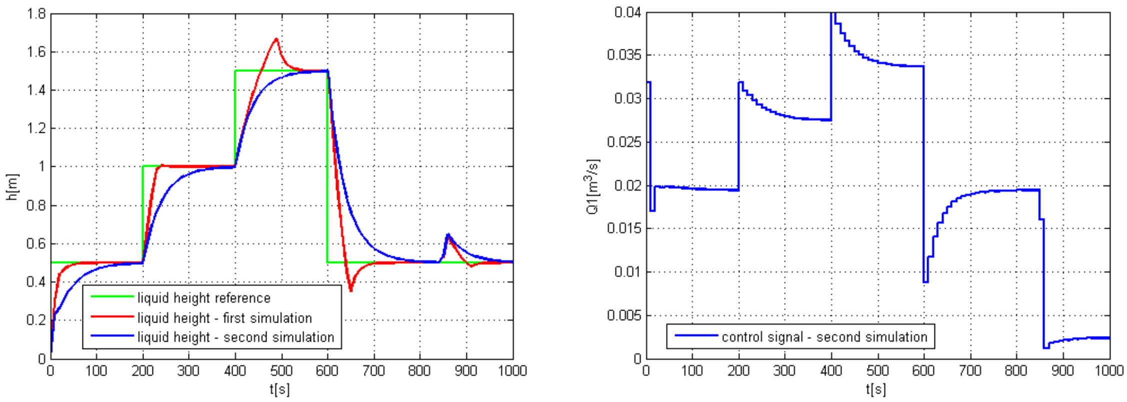

Figure 4: Second simulation results

3.3 Third simulation – GPC QP

In the third simulation the following input and output constraints have been taken into account:

- $u_{min} = -Q_{10}\ m^3 s^{-1}$,
- $u_{max} = (0{,}04 - Q_{10})m^3 s^{-1}$,

- $\Delta u_{max} = 1 \, m^3 s^{-1}$,
- $y_{min} = 0 \, m$,
- $y_{max} = 2 \, m$.

$Q_{10} = 0.0194 \; m^3 s^{-1}$ is the input flow rate in operating point corresponding to the liquid height 0.5 m. The incorporation of these bounds into the GPC QP design procedure results in such control signal that will not violate its amplitude limits, the actuator slew rate will not exceed its maximal possible value and at the same time the calculated control signal will not give rise to the output outside its operating range.

In the following the GPC QP control performances are compared to those obtained using the standard GPC. In order to achieve the fast controller response the control tuning parameters of both controllers have been set to the same values as in the first simulation in section 3.1. The simulation results are in Fig. 5. It can be seen that the GPC QP preserves the fast controller response while respecting given input/output constraints, so the calculated control signal does not need to be truncated.

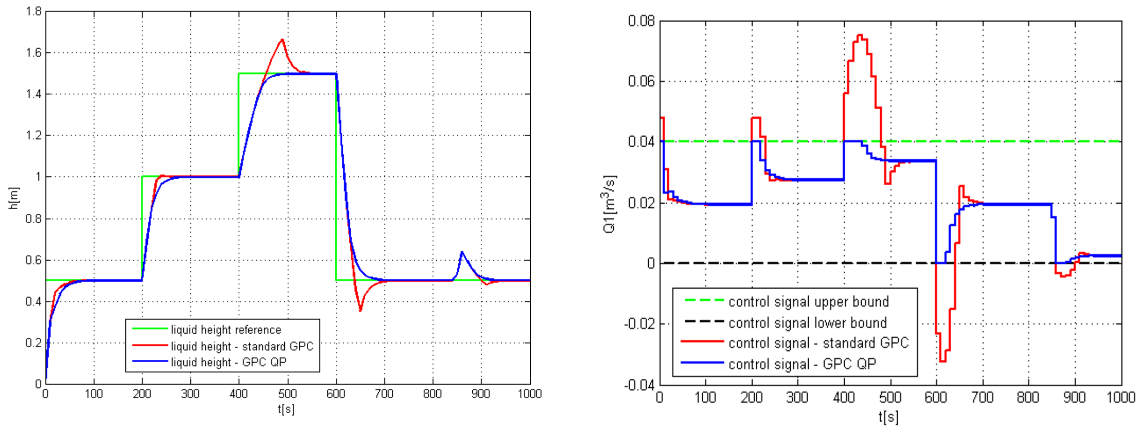

Figure 5: Third simulation results

For the liquid height reference step change from 0.5 m to 1 m the GPC QP settling time is slightly worse than that of standard GPC. On the other hand, when the reference value changes from 1 m to 1.5 m, the output response using the GPC QP controller is much better (both the settling time and the maximal overshoot) than the output response obtained using the standard GPC controller.

If the control tuning parameters will be chosen as in the second simulation, the performances of both standard GPC and GPC QP controllers are identical.

4 Experimental evaluation

The effectiveness of GPC QP controller has also been evaluated by real-time control of a cylindrical laboratory tank depicted in Fig.6. The input is the inflow servo valve opening and the output is the liquid height measured by a pressure sensor. The outflow servo valve opening has been used to generate a disturbance. The servo valves are governed by voltage 0 – 10 V. The pressure sensor range is also 0 – 10 V.

The control signal has been calculated in PC and implemented using the programmable logic controller (PLC) Simatic S7-200. For communication between PC and PLC the OPC (OLE for Process Control) communication standard has been used [3]. This standard defines methods for exchanging real-time automation data between PC-based clients using Microsoft operating systems [4]. The control scheme is shown in Fig. 7.

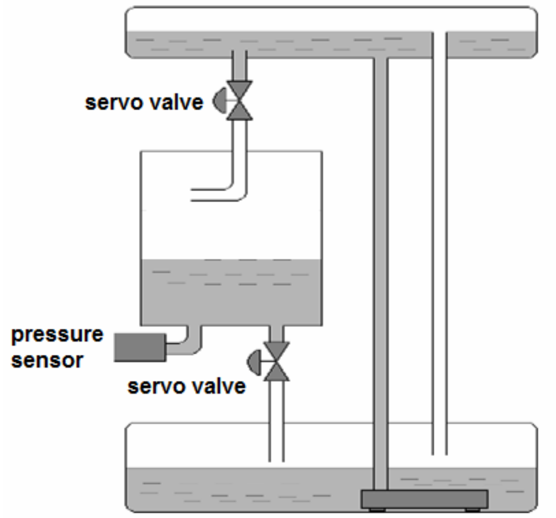

Figure 6: Cylindrical laboratory tank

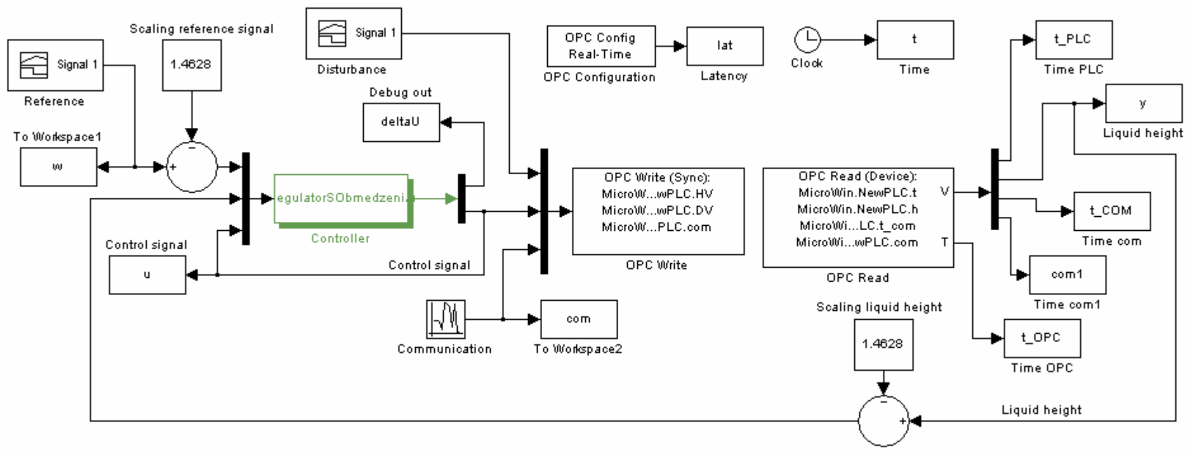

Figure 7: Control scheme

The following model of liquid height dynamics has been identified around an operating point 4.5 V with the sampling time $T_s = 1\,s$

$$G_R\left(z^{-1}\right) = \frac{-0{,}002254z^{-1} + 0{,}003291z^{-2}}{1 - 1{,}916z^{-1} + 0{,}917z^{-2}} \tag{13}$$

The control design parameters have been set for both standard GPC and GPC QP controllers to the same values

- $ph = 30,$
- $ch = 2,$
- $ro = 10,$

and the input constraints are the following

- $u_{min} = 0V$,
- $u_{max} = 10V$,
- $\Delta u_{max} = 1V$.

The real-time control results are in Fig. 8. For the liquid height reference step change from 3 V to 5.5 V the overshoot of the liquid level obtained with the GPC QP controller is considerably less then that obtained with the standard GPC. After the next liquid height reference step change to 3 V, when the standard GPC control signal does not violate the limits, the performances of both controllers are equivalent. In order to verify the regulation capabilities the disturbance in the form of the outflow valve opening decrease has been introduced at time 450 s and after 200 s the valve returned to its original position.

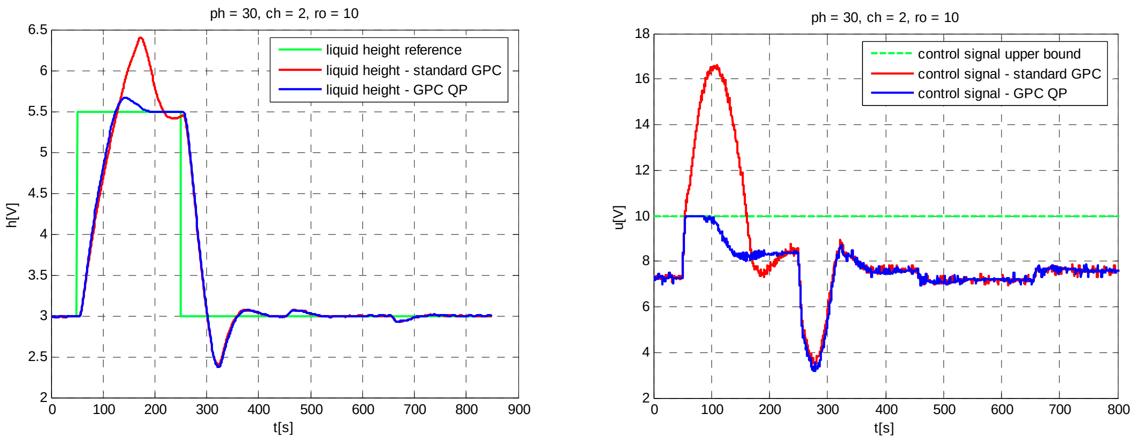

Figure 8: Real-time control results

5 Conclusion

Model predictive control represents a family of advanced control methods which are popular in both the research and industrial community. In this paper two generalized predictive control design strategies have been compared: the standard one and the modification taking into account the input/output constraints. It can be concluded from the simulations as well as from the real-time experiments that the incorporation of the process variables limits into the control design procedures leads to improvement of the control system performances although, on the other hand, the implementation of this algorithm is more complex and time-consuming. All calculations, simulations and real-time experiments have been performed in Matlab.

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