

VISUALISATION OF TRANSCENDENT TRANSFER FUNCTIONS

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Abstract

The paper is oriented to the visualisation of $F(p)$ and $F(z)$ transfer functions using a new Matlab function which enables to study the conform mapping properties to make the educational process easier than before.

Introduction

Dealing with transfer functions is very necessary from the educational point of view providing better insight into the point at issue of engineering application. Complex functions of complex variable play role both in math and in technical applications. Any transfer function can be studied as conform mapping in the theory of linear dynamic systems as the theory of digital control. When the linear system is described using PDE, then the adequate transfer functions are transcendent ones.

Mass and heat transfer

The transcendent transfer functions occur in case of mass and heat transfer modelling. The mass transfer in solid isotropic background was studied at first. The diffusivity was supposed to be unit. The mass transfer from the surface of a wall with infinite thickness to the unit depth is

$$F(p) = e^{-\sqrt{p}} \quad (1)$$

where p is the complex independent variable.

The mass transfer from a side of a tin wall with unit thickness to the opposite side is

$$F(p) = \frac{1}{\cosh(\sqrt{p})} \quad (2)$$

The mass transfer from the surface of a narrow cylinder with unit radius to its axis is

$$F(p) = \frac{1}{J_0(j\sqrt{p})} \quad (3)$$

The mass transfer from the surface of a unit sphere to its centre is

$$F(p) = \frac{\sqrt{p}}{\sinh(\sqrt{p})} \quad (4)$$

Lately the mass transfer in a flow with axial dispersion was included. The unit flow velocity, the unit tube length and the absence of radial dispersion were supposed.

Resulting transfer function is

$$F(p) = \exp\left(\frac{Pe}{2} - \sqrt{\frac{Pe}{4} + p \cdot Pe}\right) \quad (5)$$

where $Pe > 0$ is Peclet criterion of the axial dispersion.

The two limit cases are outstanding from the technical point of view. The case of infinite diffusivity is described as

$$\lim_{Pe \rightarrow 0^+} F(p) = 1 \quad (6)$$

The case of ideal flow without axial dispersion is ordinary time delay system

$$\lim_{Pe \rightarrow +\infty} F(p) = e^{-p} \quad (7)$$

The transfer functions are often seen by students as some formulas without any property. That is why we decided to prepare a tool for their visualisation, de-mysticism and understanding.

Matlab function for conform mapping visualisation

The function TRANSFERMAP.M in Matlab was created and tested for the transfer function visualisation. Both Laplace transform function $F(p)$ and Z-transform function $F(z)$ can be visualised. The first step of implementation there is a composition of complex value expression for the right part after $F(p)$ or $F(z)$ respectively. The expression consists of complex variable p or z , real or complex constants, scalar functions of complex variable, brackets and basic operators $+$, $-$, $*$, $/$, $.*$ or $./$...

The expression string is the first parameter of the visualisation function. The second parameter there is an upper constraint for absolute value of real and imaginary part of independent complex variable p or z . A positive contour level distance is the third parameter. The next parameter enables graphical contour presentation in colours, black and white or in grey scale. The last parameter specifies the dependent variable for contour plotting. We can plot the equidistant contours of real part, imaginary part, absolute value or argument of $F(p)$ or $F(z)$. The superposition of real and imaginary contours were frequently used in the old textbooks about conform mapping, complex functions and 2D potential fields. This option is also available. The source code is easy to run within Matlab system.

The result of calculation gives the standard plot for export and presentation. A complex matrix of transfer function values for alternative processing and visualisation is the side result. The source code illustrates how easy it is to construct something new and small but effective. It can be used for Matlab training and motivation for the other use.

Source code of the MATLAB function

```
function [f]=TRANSFERMAP(transfer, range, deltah, bw, matype)
% Contour Map of Given Transfer Function F(p) or F(z)
% [f]=TRANSFERMAP(transfer, range, deltah, bw, matype);
% f ..... complex transfer matrix (401,401)
% transfer .. transfer function expression with p or z variable
% range ..... -range<=Re p,Im p, Re z, Im z<=+range (range=3)
% deltah ..... contour level difference deltah>0 (deltah=0.1)
% bw ..... 0 default colors, 1 black and white, 2 grey
% matype ... 0 ... Re F and Im F together
% ..... 1 ... Re F ..... 2 ... Im F
% ..... 3 ... Abs F ..... 4 ... Arg F
% Examples:
```

```

% [f]=TRANSFERMAP('1./p',3,0.1,1,1);
% [f]=TRANSFERMAP('exp(1./z)',3,0.1,2,3);
% Initial settings
n=200; nh=100; xi=-range:range/n:+range; h=-nh*deltah:deltah:+nh*deltah;
% Transfer expression processing
xtr=transfer; xtr(findstr(transfer,'.'))=[];
if size(findstr(transfer,'p'))>0
    gtype='p';
else
    gtype='z';
end
% The main subject of the TRANSFERMAP function
[x,y]=meshgrid(xi); p=x+i*y; z=p; f=eval(transfer,'0');
% That's all folks
% Adequate contour plotting
gmsg='';
switch maptype
case 0
    gmsg='Real and imaginary part'; contour(x,y,real(f),h); hold on
    contour(x,y,imag(f),h); hold off
case 1
    gmsg='Real part'; contour(x,y,real(f),h);
case 2
    gmsg='Imaginary part'; contour(x,y,imag(f),h);
case 3
    gmsg='Absolute value'; contour(x,y,abs(f),h);
case 4
    gmsg='Argument'; contour(x,y,angle(f),h);
end
switch bw
case 0
    colormap('default')
case 1
    colormap([0 0 0]);
case 2
    colormap(grey);
end
% Final presentation
title([gmsg ' of transfer function F(' gtype ') = ' xtr]);
xlabel(['Re ' gtype]); ylabel(['Im ' gtype]);
axis equal; axis([-range +range -range +range]);

```

Examples of visualisation

The function TRANSFERMAP was used for the explanation of the space bounded mass or heat transfer. The real and imaginary part contours are plotted together to demonstrate the conform mapping in traditional style with contour distance 0.1 and in the black and white style. The imaginary contours are orthogonal to the real ones. The real axis is just one of imaginary part contours.

The infinite thickness wall transfer function with branching and one branching point is shown on *Fig.1*. The tin wall transfer function with single poles is shown on *Fig.2*. The narrow

cylinder transfer function with single poles is shown on **Fig.3**. The sphere transfer function with single poles is shown on **Fig.4**.

There is demonstrated the branching line from **Fig.1**. is split to infinite number of singular poles on **Figs. 2,3,4**.

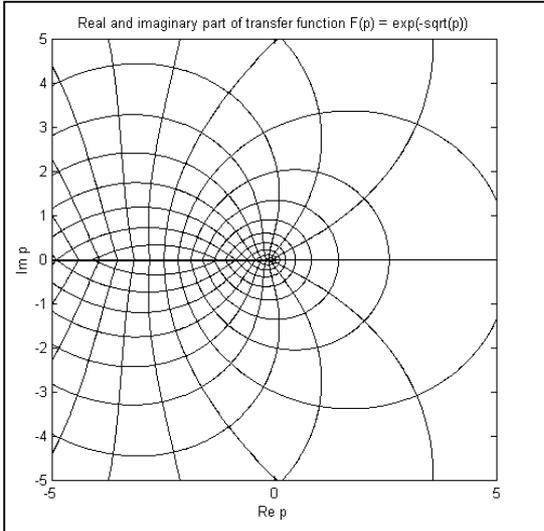


Fig. 1: Infinite wall

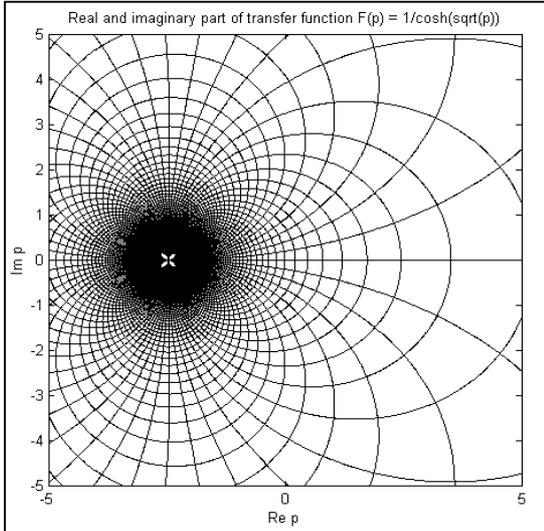


Fig 2: Thin wall

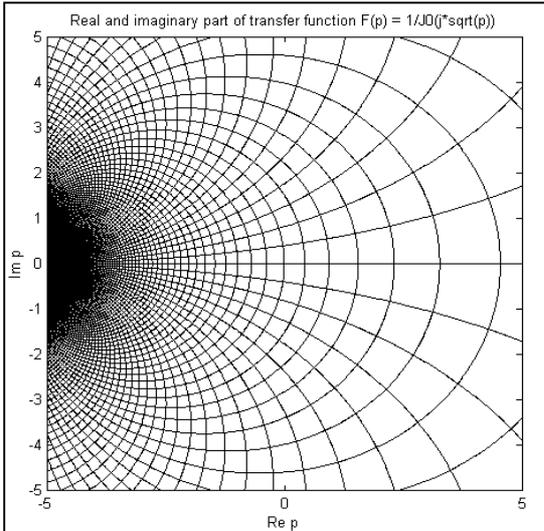


Fig. 3: Narrow cylinder

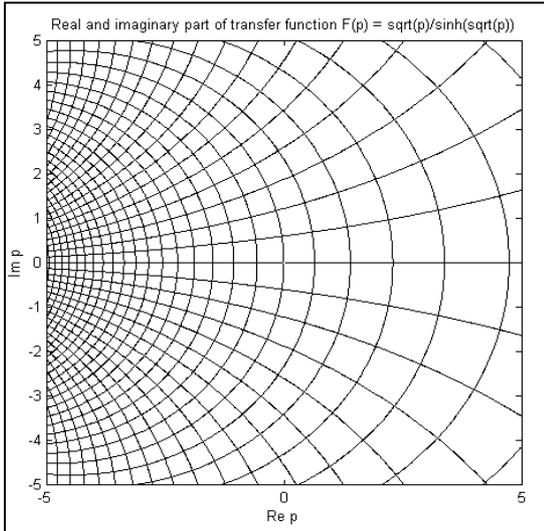


Fig 4: Unit sphere

The effect of Peclet criterion increasing is demonstrated on *Figs. 5,6,7,8*. The axial dispersion transfer function for $Pe=1$ is shown on *Fig.5*. The axial dispersion transfer function for $Pe=8$ is shown on *Fig.6*. The axial dispersion transfer function for $Pe=64$ is shown on *Fig.7*. The axial dispersion transfer function for Pe approaching infinity is shown on *Fig.8*.

The Peclet criterion influence is transparent to recognise.

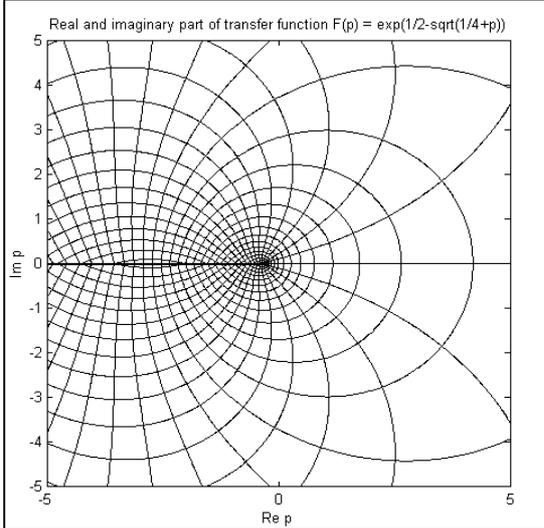


Fig 5: Axial dispersion for $Pe=1$

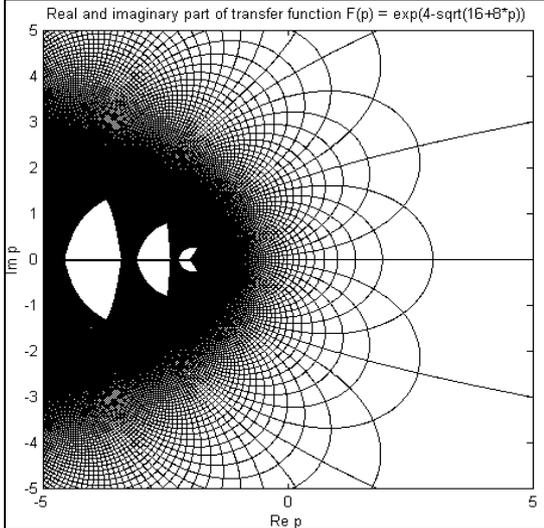


Fig 6: Axial dispersion for $Pe=8$

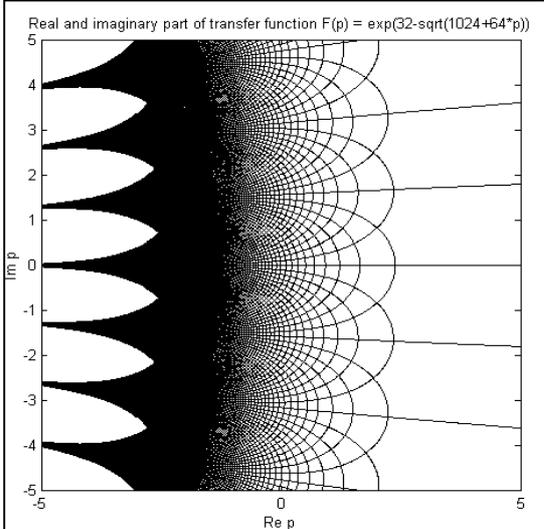


Fig 7: Axial dispersion for $Pe=64$

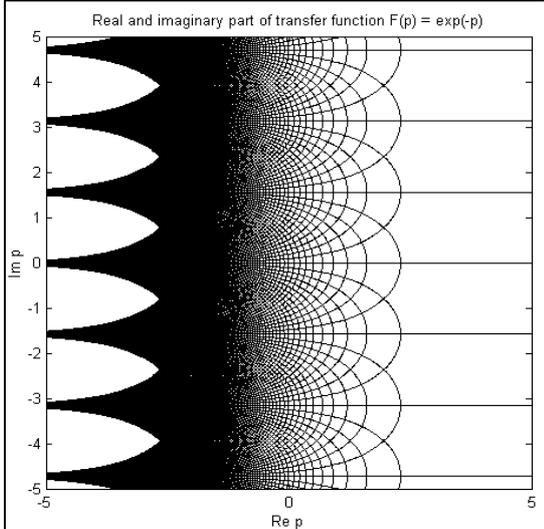


Fig 8: Time delay

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