

NEW TOOLS FOR SPHERICAL UNCERTAIN SYSTEMS IN POLYNOMIAL TOOLBOX FOR MATLAB¹

Zdenek Hurak, Michael Sebek

Center for Applied Cybernetics
Czech Technical University
Prague, Czech Republic

e-mail: hurak@control.felk.cvut.cz, m.sebek@polyx.cz

Abstract: systems with uncertain parameters are studied by one branch of robust control theory. Usually, it is assumed that the uncertain parameters are bounded by their minimum and maximum values. The uncertain parameters then form a box in the space of parameters. To provide an alternative, control theoreticians introduced the concept of spherical uncertain system. The uncertainty set is then described as an ellipsoid. Recently, two m-files were written that facilitate analysis of such systems. These are built upon Polynomial Toolbox for Matlab and extend its capabilities in the field of robust control.

Keywords: stability analysis, robustness, uncertain polynomials.

1. INTRODUCTION

One of the most serious drawbacks of standard analysis and synthesis techniques for control systems is that they rely upon the exact description of the plant. The reality is however, that such description is hardly ever available. To overcome the associated difficulties, the robust control theory developed over past few decades. One of its branches deals with situations, where the model of the plant obtained by modeling is very close to the real system, with the exception of a few physical parameters. These parameters are only known to be bounded. The most natural bounds are represented by the minimum and maximum values of the uncertain parameters, e.g. mass of the load ranging from 100 to 350 kg. In such cases the uncertainty set can be described as a box in the space of parameters. An alternative is to provide bounds on the weighted Euclidian norm of the vector of uncertain parameters. It can be shown that all the possible combinations of uncertain parameters then form an ellipsoid, hence the name – spherical uncertain systems. The authors of this paper have written two simple m-files that facilitate the analysis of such systems.

¹ This work/result/product was supported by the Ministry of Education of the Czech Republic under Project LN00B096.

2. SPHERICAL UNCERTAIN SYSTEMS

From the stability point of view, we are concerned with the characteristic polynomial of the system. Location of roots of this polynomial determines the stability of the system. But when some of the system parameters are uncertain, we obtain a family of polynomials $P = \{p(\cdot, q) : q \in Q\}$, with infinite number of members.

A family of polynomials is said to be spherical if $p(\cdot, q)$ has an independent uncertainty structure and Q is an ellipsoid.

The practical appeal of spherical uncertainties is less obvious. The most natural way to describe variations in parameters is to consider a box in the space of parameters. But in most situations, imprecision is present in setting the bounds on the parameters. For instance, an engineer may give the following description of the uncertain parameters: the coefficient of friction q_0 can vary up to 10 or 20 percent about its nominal value $q_0^* = 0.5$, and the mass q_1 can vary up to 5 or 7 percent around $q_1^* = 60$ kg. It is shown in Fig.1 that two box uncertainty sets are given – Q_{min} and Q_{max} . Then it is quite natural to approximate the true uncertainty set with an ellipse Q .

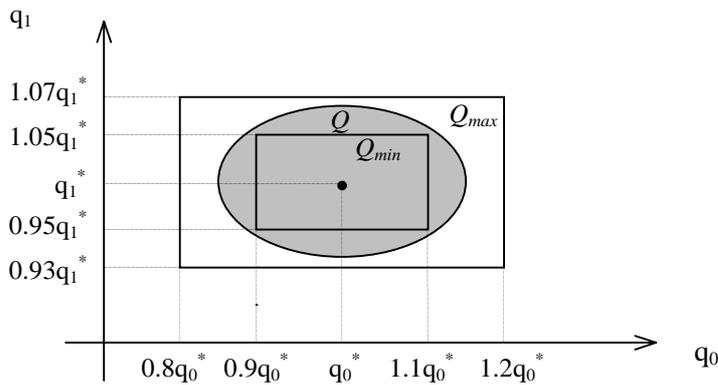


Figure 1 Box uncertainty sets Q_{min} and Q_{max} and the spherical uncertainty set Q

Systems with spherical uncertainty bounding sets are often referred to as systems with bounded l_2 norm on the parameter space. The relationship between the bounded (centered) Euclidian norm and the ellipsoid is obvious. The bounded (centered) l_2 norm on the parameter vector can be expressed as

$$\|\mathbf{q} - \mathbf{q}^*\|_{2, \mathbf{W}} \leq r$$

where \mathbf{W} is the positive-definite *weighting matrix*, $r \geq 0$ is the bound on the norm, $\mathbf{q}^* \in \mathbf{R}^k$. Then such an uncertainty set can be visualized as an ellipsoid described by

$$\left((\mathbf{q} - \mathbf{q}^*)^T \mathbf{W} (\mathbf{q} - \mathbf{q}^*) \right) \leq r^2$$

where $W = \begin{pmatrix} w_1^2 & 0 & \dots \\ 0 & w_2^2 & 0 \\ \vdots & 0 & \ddots \end{pmatrix}$. Then the ellipsoid is centered at \mathbf{q}^* and the lengths of its half-axes are given by $\frac{r}{w_1^2}, \frac{r}{w_2^2}, \dots$

3. STABILITY ANALYSIS WITH POLYNOMIAL TOOLBOX

Apart from direct computation of the roots, other stability tests are taught at introductory engineering courses, like the well-known tests of Hurwitz, Routh, Mikhailov, Nyquist and others. Nonetheless, these results are of low practical value if we need to test an infinite number of polynomials, which is the case when one or more parameters are uncertain.

Recently, new theoretical results have appeared that make the stability testing of polynomial families with spherical uncertainty structure possible, see [1] for a good overview. One of the approaches is based on a surprisingly simple but powerful concept of *zero exclusion*. With the POLYNOMIAL TOOLBOX FOR MATLAB [2] it is now possible to obtain value sets for a given range of frequencies and visually test whether the origin is included. This is illustrated in the following example.

Example 1 Consider the uncertain polynomial [1, pp.262]

$$p(s, q) = (0.5 + q_0) + (1 + q_1)s + (2 + q_2)s^2 + (4 + q_3)s^3$$

with uncertainty bound $\|q\|_{2,W} \leq 1$ and weighting matrix $W = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$,

$$\text{ie., } 2q_0^2 + 5q_1^2 + 3q_2^2 + 1q_3^2 \leq 1^2$$

Use the zero exclusion principle to test graphically the robust stability of the given uncertain polynomial. First, let's transform the given polynomial into the centered form

$$p(s, q) = 0.5 + s + 2s^2 + 4s^3 + \sum_{i=0}^3 q_i s^i$$

with the uncertainty bound unchanged. Now type

```
p0 = 0.5+s+2*s^2+4*s^3; W=[2,5,3,1]; r = 1; omega = 0:.05:2;
```

The graphical output is generated by

```
spherplot(p0,omega,r,W)
```

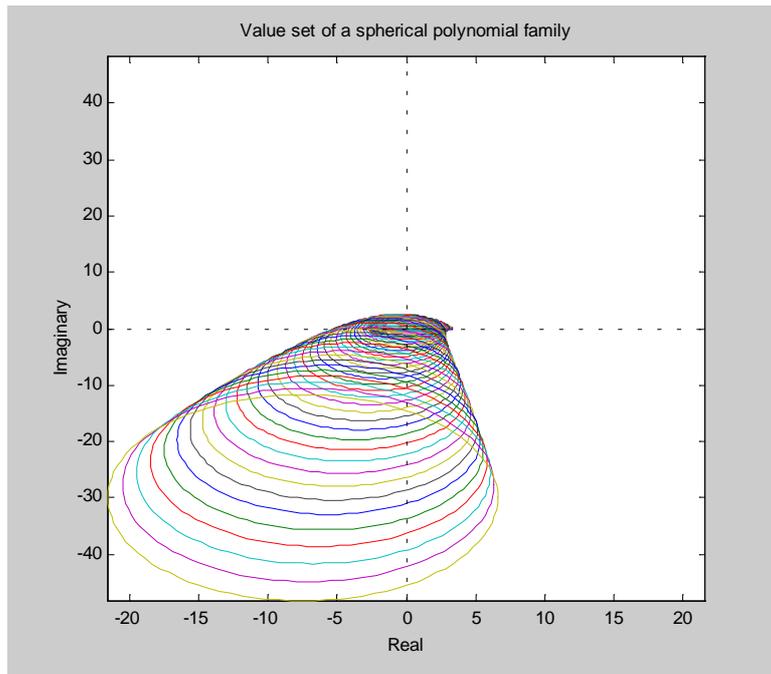


Figure 2 Value sets for a polynomial family with spherical uncertainty bounding set.

It can be clearly seen that the origin is included in the value sets, so we conclude that the given polynomial is not robustly stable.

4. ROBUSTNESS MARGIN WITH POLYNOMIAL TOOLBOX

Example 2 Consider the nominal polynomial [3], pp.147:

$$p0 = 433.5 + 667.25*s + 502.25*s^2 + 251.25*s^3 + 80.25*s^4 + 14*s^5 + s^6;$$

The diagonal entries of the weighting matrix are given

$$weight = [43.5, 33.36, 25.137, 15.075, 5.6175, 1.4, 0.1].^2;$$

The vector of frequencies is chosen as

$$omega = 0:0.01:5;$$

To compute the maximum norm on the uncertainty vector just type

$$tsyp2(p0,omega,weight)$$

$$ans = 2.8312$$

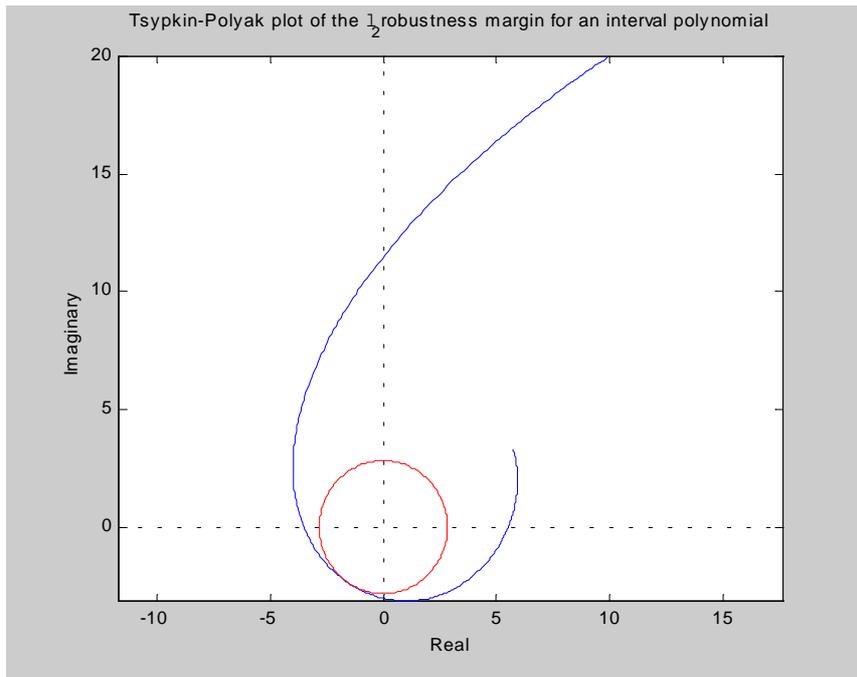


Figure 3 Tsyarkin-Polyak locus (blue) determining the bound on l_2 norm (red) of the uncertain parameter vector.

Now it is possible to check the result invoking the graphical routine SPHERPLOT and visually test for *zero exclusion condition* violation.

```
spherplot(p0,0:0.01:3,2.8312,weight)
```

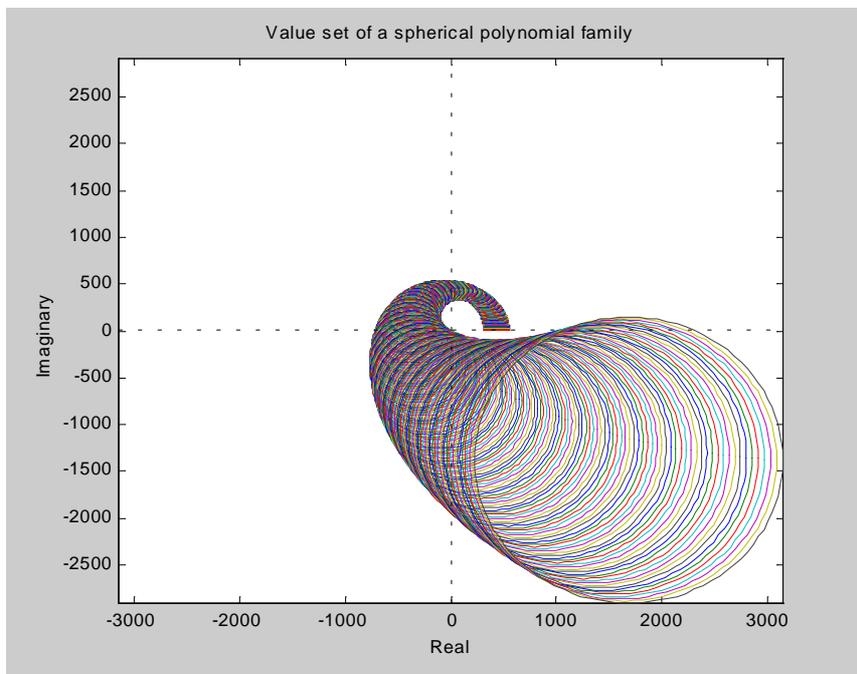


Figure 4 Value sets for maximum l_2 norm of the uncertain parameter that still preserves stability.

5. CONCLUSION

Two Matlab based routines were presented that facilitate the stability analysis of polynomial families with spherical uncertainty structure. Even though the practical appeal of such uncertainty description is less obvious than for the usual *box* uncertainty structure, they can be considered as an alternative approach. Moreover, recent theoretical achievements hint that some calculations might be easier with ellipsoid representation of a value set than with a polygon representation. Anyway, the two m-files can be downloaded freely at <http://www.polyx.com> and experimented with.

REFERENCES

- [1] Barmish, B.R. (1994). *New Tools for Robustness of Linear Systems*, Macmillan Publishing Company.
- [2] Kwakernaak, H., M. Sebek (2000). Polynomial Toolbox 2.0 for use with Matlab 5, Polyx Ltd. <http://www.polyx.com>
- [3] Bhattacharyya, S.P., Chapellat, H., Keel, L.H. (1995). *Robust Control: The Parametric Approach*, Prentice Hall .