

# INTELLIGENT ADAPTIVE CONTROL

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## Abstract

The paper is concerned with the intelligent control of linear plant using multiple models. The control algorithm uses a set of models and a switching scheme. The algorithm is starting with no apriori knowledge about the plant. Multiple models used for switching are built during the process using the least-squares identification modified with directional forgetting. New model can be used effectively later whenever the parameters take the original values.

## Key Words

Adaptive control, Multiple model, Simulink, Least-squares

## 1. Introduction

Biological systems have the abilities to learn, to adapt and to recognize to cope with different conditions. They are able to create several behaviors for different situation and choose the most appropriate for given situation. Industrial systems are also characterized with several multiple operating modes. The changes among operating modes are usually abrupt. These systems are usually described by multiple models associated with the different operating modes. The use of multiple models is not a new idea. In fact, multiple Kalman filters were proposed in 70's to improve the accuracy of the state estimate in the control problem. The architecture of the multiple models, the switching and tuning methodology was introduced in [1]. The qualitative description of the approach may be found in [2] and [3]. Other switching schemes can be found at [4],[5]. During the last years the multiple-model approach has become very popular and widely applied for estimation and control of stochastic systems under different kinds of uncertainty - unknown model structure or parameters. Multiple model control is of great importance to problems such as fault tolerant control [6],[7] or noise identification [8]. Multiple models can be also used to control the nonlinear systems. Local model networks represent a nonlinear dynamical system by set of locally valid submodels across the operating range [9]. This method uses model weighting instead of switching between the separate controllers. The major drawback of this algorithm is an assumption of having off-line identified fixed models. This assumption involves analysis of the plant dynamics. The off-line identification can be avoided using a large amount of models [2]. While traditional adaptive control is suitable for slow changing parameters, it can hardly cope with the complex systems with several operating points because it takes some time to learn new parameters and change the parameters of the controller. This type of control often leads to poor control performance. The idea of the multiple model control is to off-line identify the parameters of the operating modes. The overall system is then controlled by switching between the controllers based on the different operating modes. To guarantee the stability of the overall system the identification with the least-squares algorithm is added. Our approach combines the conventional adaptive control with multiple model control and does not require the off-line identification.

## 2. Adaptive control

In common sense, "to adapt" means to change a behavior according to conditions. Intuitively, an adaptive controller is an intelligent controller that can modify its behavior in response to the variations in dynamics of the process and the character of the disturbance. The development of the adaptive control started in the 50's with the aim of developing adaptive flight control. In the 60's and 70's

algorithms with guaranteed stability, convergence and robustness properties were developed. Plant nonlinearities can be found in the most of processes from flight control to process control. The self-tuning has been developed to cope with these nonlinearities. They can be also used when the parameters of the system are uncertain. The self-tuning control consist of two operations: model building via identification and controller design using identified model. Identification represents an on-line estimation of the process parameters using least-squares or projection algorithm. Sometimes the self-tuning algorithm can be simplified by reparameterizing and directly estimating the controller parameters. The least-squares method is commonly used in system identification. Its principle is that the unknown parameters of a mathematical model should be chosen by minimizing of the square of the difference between the actually observed and analytically predicted output values. The RLS algorithm has several modifications suitable for specific applications. For instance, RLS with exponential forgetting algorithm which exponentially weights the passed data so that it becomes less and less significant as time progresses.

### 3. Multiple Models

Multiple models method is a powerful adaptive approach. It has received a great deal of attention in recent years due to its unique power in handling problems with structural and parametric uncertainties or changes. The experience showed that conventional adaptive schemes control performs well in many cases. These schemes are appropriate for systems with slow varying dynamics, systems where the initial estimates are close to their real values. In other cases the conventional adaptive schemes can be quite poor. If the plant is subjected to sudden changes of parameters or the magnitude of disturbance, the identification can take a considerable time. Such difficulties have lead to the emergency of the new approach. Instead of using single model several fixed and adaptive models are used.

#### Adaptive models

The adaptive control consists of estimation of parameters of the plant using the identification model and forming a control input based on the estimated parameters. If the parameter uncertainty is huge, the response can be poor. To improve the response several adaptive models with different initial parameter estimates can be added. The model with the smallest error is then chosen for the control. If the parameters of the plant change, the change must be recognized and the estimates have to relocate to new values.

#### Fixed models

Fixed models can be used to represent the operating modes that are precisely known. Fixed models require less computation than adaptive models. Fixed models can tolerate small errors in modeling due to their inherent robustness. A large set of fixed models is needed if there is a huge uncertainty of plant parameters. The fixed models can provide speed whenever its parameters are close enough to those of the process while adaptive models can provide the accuracy.

#### Switching

The control strategy is to determine the most appropriate model for the current operating condition at every instant and activate the according controller. This control structure as shown on Fig. 1 is based on  $N$  models which have been determined at various operating point. A controller is then designed for each model. A supervisor then compares the output errors for each of the model and chooses the closest model according the criterion  $J_i(t)$

$$J_i(t) = \alpha e_i^2(t) + \beta \int_0^t e^{-\lambda(t-\tau)} e_i^2(\tau) d\tau \quad (1)$$

where the constants  $\alpha > 0$  and  $\beta > 0$  reflects the importance of the current and past values and the constant  $\lambda > 0$  is weighting factor which will influence the speed of the supervisor. Here  $e_i(t)$  is the output error for  $i$  model.

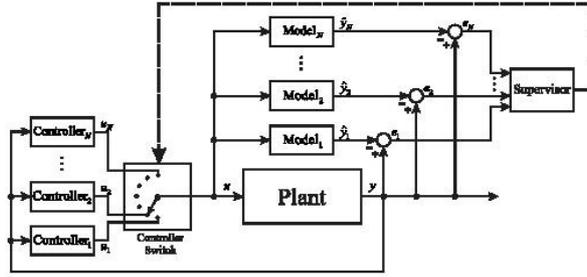


Fig. 1 Multiple model scheme

#### 4. Learning

The conventional adaptive control uses only a single model. Whenever the parameters of the system change the old values of identified parameters  $\theta(k)$  are forgotten. If the parameters recover their original values, the even if the parameters were successfully identified in the past. The idea of this approach is not to forget the identified models so they can be used later. The parameters of the system are identified using RLS method. In this paper we assume the model of the second order given by

$$G(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad (2)$$

The recursive least-squares method with exponential forgetting modifies the basic RLS updating algorithm to weigh new information more heavily. Loss function is given by

$$J = \sum_{i=k_0}^k \left[ \varphi^{k-i} \left( y(k) - \hat{\theta}^T(k) \Phi(k-1) \right) \right]^2 \quad (3)$$

where  $\varphi$  is an exponential forgetting factor from the range (0;1),

$\hat{\theta}(k) = [a_1 \ a_2 \ b_1 \ b_2]$  and  $\Phi^T(k) = [-y(k-1), -y(k-2), u(k-1), y(k-2)]$  are vector of parameter estimates and a regression vector [10].

The algorithm of the exponential forgetting was adjusted with directional forgetting which forgets only in the direction where new data are obtained [11]. The vector of parameter estimates is according to recursive relation

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{C(k-1)\Phi(k-1)}{1 + \xi(k-1)} \hat{e}(k-1) \quad (4)$$

where

$$\xi(k) = \Phi^T(k-1)C(k-1)\Phi(k-1) \quad (5)$$

is the auxiliary scalar and

$$\hat{e}(k) = y(k) - \hat{\theta}^T(k)\Phi(k-1) \quad (6)$$

is the prediction error. If  $\xi(k) > 0$ , the square covariance matrix  $C(k)$  is made current by relation

$$C(k) = C(k-1) - \frac{C(k-1)\Phi(k-1)\Phi^T(k-1)C(k-1)}{\varepsilon^{-1}(k) + \xi(k-1)} \quad (7)$$

where

$$\varepsilon(k) = \varphi(k) - \frac{1 - \varphi(k)}{\xi(k-1)} \quad (8)$$

If  $\xi(k-1) = 0$ , then

$$C(k) = C(k-1) \quad (9)$$

The value of adaptive directional forgetting factor  $\varphi(k)$  is then calculated for each sampling period according to relation

$$\varphi(k) = \left\{ 1 + (1 + \rho) \left[ \ln(1 + \xi(k-1)) \right] + \left[ \frac{(\nu(k-1) + 1)\eta(k-1)}{1 + \xi(k-1) + \eta(k-1)} - 1 \right] \frac{\xi(k-1)}{1 + \xi(k-1)} \right\}^{-1} \quad (10)$$

where

$$\eta(k) = \frac{\hat{e}^2(k)}{\lambda(k)} \quad (11)$$

$$\nu(k) = \varphi(k) [\nu(k-1) + 1] \quad (12)$$

$$\lambda(k) = \varphi(k) \left[ \lambda(k-1) + \frac{\hat{e}^2(k-1)}{1 + \xi(k-1)} \right] \quad (13)$$

are auxiliary variables.

The changes of the parameters of the system are connected with the changes of estimates  $\hat{\theta}(k)$ , so they can be identified by measuring  $|\hat{\theta}(k) - \hat{\theta}(k-1)|$ . Whenever the system switches from one mode to another, estimates vector  $\hat{\theta}(k-1)$  that is considerably different from those that have been included in the model bank would create a new model  $M_{m+1}$ . The measure of similarity between the adaptive model  $\hat{\theta}_a(k-1)$  and the closest model of the model bank  $\hat{\theta}_c$

$$\hat{\theta}_c = \arg \min \left\| \hat{\theta}_a(k-1) - \hat{\theta}_i \right\|, \quad i = [1, m] \quad (14)$$

is defined as a normalized distance between corresponding parameter vectors [12].

$$\left\| \hat{\theta}_a(k-1) - \hat{\theta}_c \right\| / \left\| \hat{\theta}_c \right\| < e \quad (15)$$

It defines a ball in the parameter space within which the model parameters are considered similar. As new operating modes appear, the number of fixed models gradually increases.

## 5. Switching scheme

The output of each model is computed using past values of the output, input and parameters of each of the models  $\hat{\theta}_i$

$$\begin{aligned} \hat{y}_i(k) &= \hat{\theta}_i^T \varphi(k) \\ \varphi^T(k) &= [y(k-1), y(k-2), u(k), u(k-1)] \end{aligned} \quad (16)$$

where  $\hat{y}_i$  is the output of  $i$  model and  $y(k)$  is the measured output of the plant. At every instant the error between output predicted by different models and actual plant output is computed and the best model is chosen using the criterion

$$J_i(k) = \alpha e_i^2(k) + \beta \sum_{j=1}^M \exp(-\lambda j) e_i^2(k-j) \quad (17)$$

The controller corresponding to the chosen model is used. If the system exhibits rapid switching between controllers, the degraded performance could occur. The switching performance can be improved by introducing a waiting period  $T_w$  after every switch to prevent the arbitrary switching. To counter the undesirable effect of switching solution we applied here a hysteresis algorithm [13]. Suppose that controller  $C_l$  is being used at the step  $t$  and  $C_k$  is a winning model at the step  $t$

$$J_k(t) = \min_i (J_i(t)).$$

If  $J_l(t) \leq J_k(t) + \delta$  the controller  $C_l$  will be retained, and switched to controller  $C_k$  otherwise. Here  $\delta > 0$  is the hysteresis constant. This also prevents switching to an adaptive model when a fixed model is close enough to actual parameters of the system.

## 6. Simulation Example

The plant to be control is piecewise LTI. It consists of 3 operating modes with transfer functions:

$$\begin{aligned}
 1. \quad \frac{Y}{U} &= \frac{1}{s^2 + s - 2} \\
 2. \quad \frac{Y}{U} &= \frac{1}{s^2 - s + 2} \\
 3. \quad \frac{Y}{U} &= \frac{1}{s^2 + 4s + 4}
 \end{aligned} \tag{18}$$

The mode is randomly switched every 18 seconds. RLS method with directional forgetting is used for online parameter identification of the discrete linear model. The sampling period was set to 10 Hz. The Simulink scheme can be seen in Fig. 3.

The controller is designed to follow a stable closed loop characteristic polynomial

$$A(s) = s^2 + 4.2s + 9 \tag{19}$$

The structure of the digital controller can be seen in Fig. 2. The controller parameter  $\beta$  and polynomials  $P(z^{-1})$ ,  $Q(z^{-1})$  are computed using the diophantine equation

$$A(z^{-1})P(z^{-1}) + B(z^{-1})(Q(z^{-1}) + \beta) = D(z^{-1}) \tag{20}$$

where

$$\begin{aligned}
 P(z^{-1}) &= (1 - z^{-1})(1 + \gamma z^{-1}) \\
 Q(z^{-1}) &= (1 - z^{-1})(q_0 - q_2 z^{-1}) \\
 D(z^{-1}) &= 1 + d_1 z^{-1} + d_2 z^{-2}
 \end{aligned} \tag{21}$$

and  $A(z^{-1})$ ,  $B(z^{-1})$  are the polynomials of plant.

The control objective is to follow a reference signal with amplitude 1 and period 10 seconds.

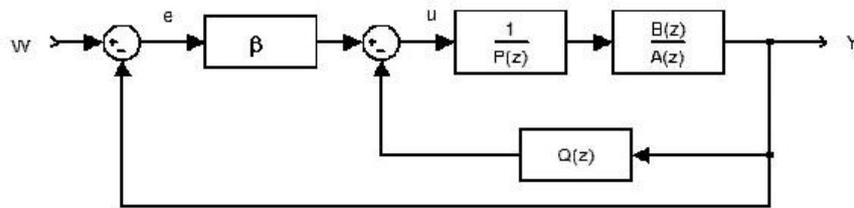


Fig. 2 The control scheme

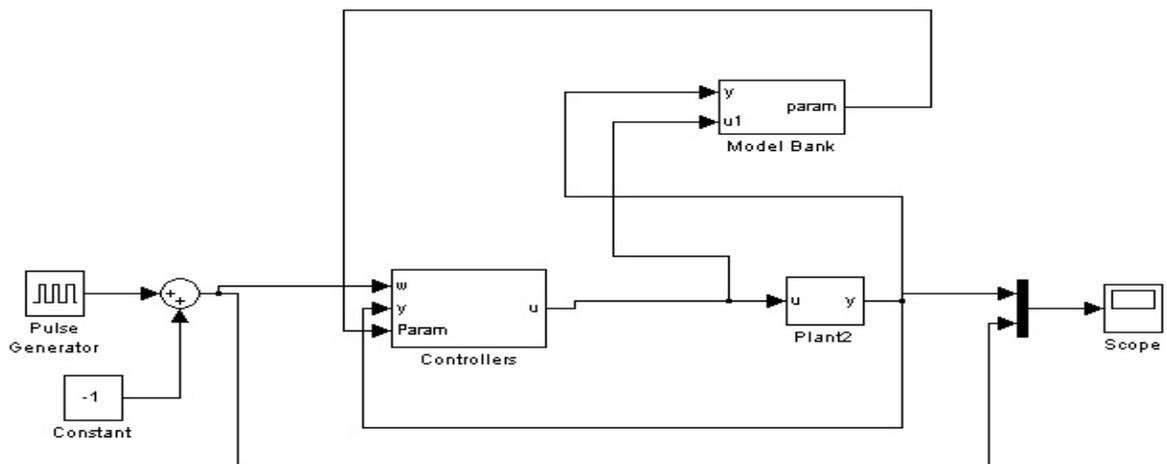


Fig. 3 The Simulink scheme

The models are automatically created at the time 36s, 54s and 108s. The results of the control method and sequence of operating modes can be seen on Fig. 4 a 5. The improvements in the performance can be seen after all the operating modes are identified and added to the set of models. The control changes from pure adaptive control to multiple model control with only fixed models. Whenever new operating regime occurs, it is added to the set of models so it can be used later.

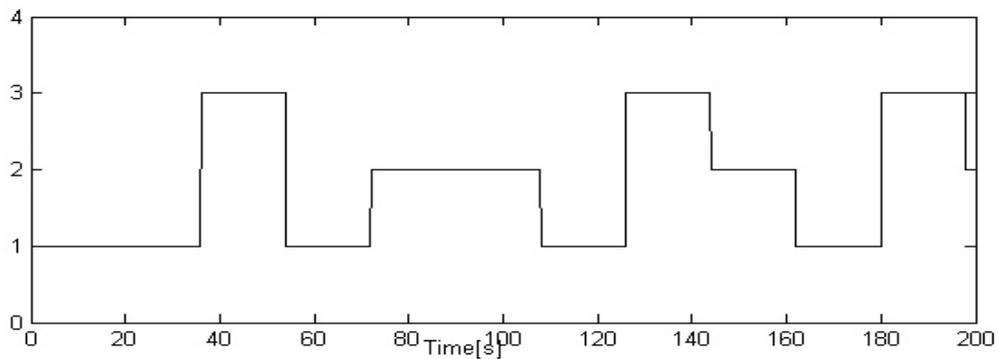


Fig. 4 Operating modes

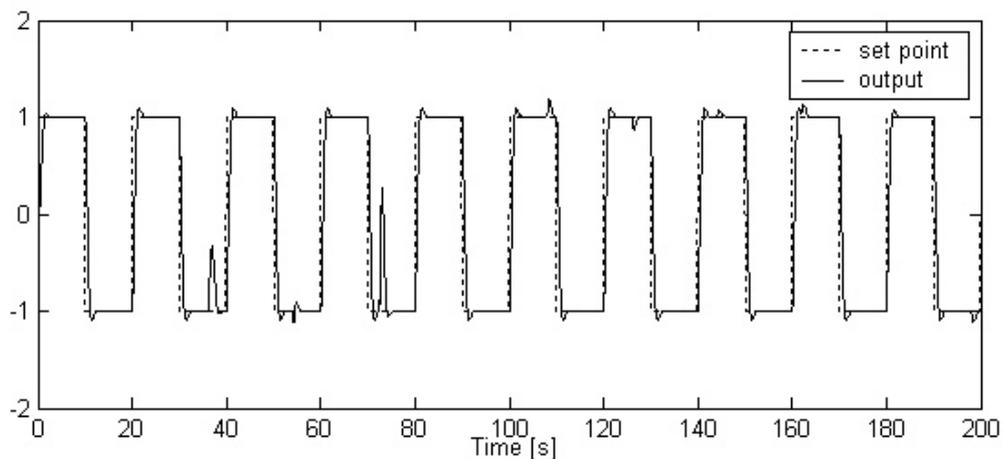


Fig. 5 Simulation results

## 6. Conclusion

The control of dynamical systems with rapidly changing environments is considered in this paper. Multiple fixed models are used to represent different operating regimes of the plant. After the end of unknown operating mode a new model is added to the set of models so it can be used later. It has been shown through simulations that response after sudden parameter change can be improved by switching to these fixed models.

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