VARIATION APPROACH TO INVARIANT RECOGNITION OF BINARY IMAGES

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Abstract

Binary image of individual n-dimensional object is an information source for object recognition. The properties extracted from given binary image should be invariant to translation (T), scaling (S), and rotation (R) of the original pattern, object, or image, respectively. There are many possibilities how to realize TSR invariant properties of n-dimensional binary images. The translation invariance can be achieved by using n-dimensional Fourier transform and amplitude spectrum, which is trivial. The rotation of original will cause rotation of Fourier spectrum. Thus the rotation invariance is based on envelopes, which are generated by rotation of Fourier spectrum. The resulting envelopes can be rescaled to normalized forms which are TSR invariant. The recognition system uses TSR invariant envelopes as non-linear preprocessing for proposed variation recognizer. Standard PCA technique is used as referential method. All the programs are realized in Matlab environment.

1 Introduction

The translation of object in $\mathbb{R}^n$ must not complicate the functionality of any recognition system, which is called translation invariant. Similar objects of various size are often supposed to be identic, which motivates the scaling invariance. The special applications require the rotation invariance of recognition system. Grain particle recognition, categorization, and counting are typical TSR invariant tasks, while the character recognition must not be rotation invariant due to pairs: WM, 25, 69, E3.

2 Preliminaries

Real objects are studied through their 1D, 2D or 3D images. They can be transformed to gray form, proceeded, filtered, denoised, or enhanced and then converted to binary form by segmentation or thresholding. The definitions of basic image terms are necessary.

- Let $n \in \mathbb{N}$ be *image dimension*, $N \in \mathbb{N}$ be hypercube size.
- *Image volume*: $\mathrm{V}(\mathrm{f}) = \int\limits_{\mathbb{R}^n} \mathrm{f}(\overline{x})\,\mathrm{d}\overline{x}$.
- *Hypercube, discrete hypercube*: $\mathbb{C}_{n,N} = [0; N]^n, \mathbb{K}_{n,N} = \{0; \ldots; N-1\}^n$.
- Sets of non-zero finite gray and binary images: $\mathbb{G}^*_{n,N}, \mathbb{B}^*_{n,N}$.
- Sets of non-zero discrete gray and binary images: $\mathbb{G}^+_{n,N}, \mathbb{B}^+_{n,N}$.
- Let $\mathbb{W}_n = \{\mathrm{f} : \mathbb{R}^n \to \mathbb{C}, \int\limits_{\mathbb{R}^n} |\mathrm{f}(\overline{x})|\,\mathrm{d}\overline{x} < +\infty\}$, $\mathbb{D}_{n,N} = \{\mathrm{f} : \mathbb{K}_{n,N} \to \mathbb{C} \setminus \{\infty\}\}$.
  Then $\mathbb{B}^*_{n,N} \subset \mathbb{G}^*_{n,N} \subset \mathbb{W}_n$ and $\mathbb{B}^+_{n,N} \subset \mathbb{G}^+_{n,N} \subset \mathbb{D}_{n,N}$.
- *Fourier transform* is then defined as $\mathcal{F}_n : \mathbb{W}_n \to \mathbb{W}_n$ satisfying

$$\mathcal{F}_n\{\mathrm{f}(\overline{x})\} = \mathrm{F}_n(\overline{\omega}) = \int\limits_{\mathbb{R}^n} \mathrm{f}(\overline{x})\mathrm{e}^{-i\,\overline{\omega}'\,\overline{x}}\,\mathrm{d}\overline{x}. \quad (1)$$

- *Discrete Fourier transform* is defined as $\mathcal{D}_{n,N} : \mathbb{D}_{n,N} \to \mathbb{D}_{n,N}$ satisfying

$$\mathcal{D}_{n,N}\{\mathrm{f}(\overline{x})\} = \mathrm{F}_{n,N}(\overline{\omega}) = \sum_{x_1=0}^{N-1} \cdots \sum_{x_n=0}^{N-1} \mathrm{f}(\overline{x})\mathrm{e}^{-\frac{2\pi i}{N}\overline{\omega}'\,\overline{x}}. \tag{2}$$

3 TSR invariant envelopes: Basic facts

Definition 1: *Let* $\mathrm{f} \in \mathbb{W}_n, \overline{x}_0 \in \mathbb{R}^n$. *The function* $\mathrm{g} \in \mathbb{W}_n$ *is called* ***translation*** *of* f *when* $\mathrm{g}(\overline{x}) = \mathrm{f}(\overline{x} - \overline{x}_0)$.

Theorem 1: *Let* $\mathrm{g} \in \mathbb{W}_n$ *be any translation of* $\mathrm{f} \in \mathbb{W}_n$. *Then* $|\mathrm{G}(\overline{\omega})| = |\mathrm{F}(\overline{\omega})|$ *for* $\overline{\omega} \in \mathbb{R}^n$.

Definition 2: *Let* $\mathrm{f} \in \mathbb{G}^*_{n,N}, \overline{0} \in \mathbb{R}^n$. *Then function* $\Phi_\mathrm{f} : \mathbb{R}^n \to \mathbb{R}^+_0$ *is called* ***normalized spectrum*** *of non-zero, finite gray image* f *when*

$$\Phi_\mathrm{f}(\overline{\omega}) = \left|\frac{\mathrm{F}(\overline{\omega})}{\mathrm{F}(\overline{0})}\right|. \tag{3}$$

Definition 3: *Let* $\mathrm{f} \in \mathbb{W}_n, \overline{x}_0 \in \mathbb{R}^n$. *Let* $\mathsf{Q} \in \mathbb{R}^{n\times n}$ *be orthogonal matrix. Function* $\mathrm{g} \in \mathbb{W}_n$ *is called* ***general rotation*** *of* f *when* $\mathrm{g}(\overline{x}) = \mathrm{f}(\mathsf{Q}\,\overline{x} - \overline{x}_0)$.

Theorem 2: *Let* $\mathrm{f} \in \mathbb{W}_n$. *Let* $\mathrm{g} \in \mathbb{W}_n$ *be any general rotation of* f *with matrix* Q. *Then* $|\mathrm{G}(\overline{\omega})| = |\mathrm{F}(\mathsf{Q}\,\overline{\omega})|$ *for* $\overline{\omega} \in \mathbb{R}^n$.

Definition 4: *Let* $\mathrm{f} \in \mathbb{G}^*_{n,N}, \omega \geq 0$. *Then function* $\mathrm{L}_\mathrm{f} : \mathbb{R}^+_0 \to [0;1]$ *is called* ***lower envelope*** *when*

$$\mathrm{L}_\mathrm{f}(\omega) = \min_{\|\overline{\omega}\|=\omega} \Phi_\mathrm{f}(\overline{\omega}) \tag{4}$$

and function $\mathrm{U}_\mathrm{f} : \mathbb{R}^+_0 \to [0;1]$ *is called* ***upper envelope*** *when*

$$\mathrm{U}_\mathrm{f}(\omega) = \max_{\|\overline{\omega}\|=\omega} \Phi_\mathrm{f}(\overline{\omega}). \tag{5}$$

Theorem 3: *Let* $\mathrm{f} \in \mathbb{G}^*_{n,N}$. *Let* $\mathrm{g} \in \mathbb{G}^*_{n,N}$ *be any general rotation of* f. *Then* $\mathrm{L}_\mathrm{f}(\omega) = \mathrm{L}_\mathrm{g}(\omega)$ *and* $\mathrm{U}_\mathrm{f}(\omega) = \mathrm{U}_\mathrm{g}(\omega)$ *for* $\omega \in \mathbb{R}^+_0$.

Definition 5: *Let* $\overline{x} \in \mathbb{R}^n, a \in \mathbb{R}^+$. *Let* $\mathrm{f}, \mathrm{g} \in \mathbb{W}_n$. *Then function* $\mathrm{g}(\overline{x}) = \mathrm{f}\left(\frac{\overline{x}}{a}\right)$ *is called* ***scaling*** *of* f *with scale* a.

Theorem 4: *Let* $\mathrm{g} \in \mathbb{W}_n$ *be scaling of* $\mathrm{f} \in \mathbb{W}_n$ *with scale* $a > 0$. *Then* $\mathrm{G}(\overline{\omega}) = a^n\,\mathrm{F}\left(a\,\overline{\omega}\right)$.

Definition 6: *Let* $\mathcal{T}$ *be set of continuous functions* $\mathrm{t} : \mathbb{R} \to \mathbb{R}$. *The* ***space of envelopes*** *is defined as* $\mathcal{E} = \{\mathrm{e} : \mathbb{R}^+_0 \to [0;1] \mid \mathrm{e}\,(0) = 1,\, \mathrm{e} \in \mathcal{T}\}$.

Definition 7: *Let* $\mathrm{r} : \mathcal{E} \to \mathbb{R}^+$ *be a function satisfying*

$$\forall\, \alpha \in \mathbb{R}^+ \quad \forall\, \omega \in \mathbb{R}^+_0 :\ \mathrm{r}\left(\mathrm{L}_\mathrm{f}\left(\frac{\omega}{\alpha}\right)\right) = \alpha\,\mathrm{r}\left(\mathrm{L}_\mathrm{f}(\omega)\right), \tag{6}$$

where L_f *is lower envelope of* $\mathrm{f} \in \mathbb{G}^*_{n,N}$. *Then* $\omega^* = \mathrm{r}(\mathrm{L}_\mathrm{f})$ *is called* ***referential point*** *of* L_f.

Theorem 5: *Let* $\lambda = \frac{1}{2}$. *Let* $\mathrm{h}(x) = \min\{\omega \in \mathbb{R}^+, \mathrm{L_f}(\omega) \leq x\}$. *When* $\mathrm{h}(\lambda)$ *exists, then* $\omega^* = \mathrm{h}(\lambda) = \mathrm{r}(\mathrm{L_f}(\omega))$ *satisfies the referential point definition.*

Definition 8: *Let* $\mathrm{L_f}, \mathrm{U_f}$ *be envelopes of* $\mathrm{f} \in \mathbb{G}^*_{n,N}$. *Let* $\omega^* = \mathrm{r}(\mathrm{L_f}) > 0$ *be referential point of lower envelope* $\mathrm{L_f}$. *Then functions*

$$\mathrm{L}^*_\mathrm{f}(\omega) = \mathrm{L_f}(\omega^*\,\omega), \quad \mathrm{U}^*_\mathrm{f}(\omega) = \mathrm{U_f}(\omega^*\,\omega) \tag{7}$$

are called ***relativized lower and upper envelope*** *of* f*, respectively.*

Theorem 6: *Let* $\mathrm{g} \in \mathbb{G}^*_{n,N}$ *be scaling of* $\mathrm{f} \in \mathbb{G}^*_{n,N}$. *Then* $\mathrm{L}^*_\mathrm{f}(\omega) = \mathrm{L}^*_\mathrm{g}(\omega)$ *and* $\mathrm{U}^*_\mathrm{f}(\omega) = \mathrm{U}^*_\mathrm{g}(\omega)$.

Theorem 7: *Relativized envelopes* $\mathrm{L}^*_\mathrm{f}(\omega), \mathrm{U}^*_\mathrm{f}(\omega)$ *are TSR invariant.*

The TSR invariant envelopes $\mathrm{L}^*_\mathrm{f}(\omega), \mathrm{U}^*_\mathrm{f}(\omega)$ were derived for non-zero finite gray image $\mathrm{f} \in \mathbb{G}^*_{n,N}$ but will be used for binary image recognition.

4 Application to discrete binary 2D image

Discrete binary 2D image from $\mathbb{B}^+_{2,N}$ is a typical product of segmentation or thresholding of discrete gray 2D image from $\mathbb{G}^+_{2,N}$, which is traditional "gray photo" of size $N \times N$. The theoretical results of previous chapters were obtained for $\mathbb{G}^*_{n,N}$. Thus, they are valid for $\mathbb{B}^*_{2,N}$. But any discrete image from $\mathbb{B}^+_{2,N}$ is only an approximation of image from $\mathbb{B}^*_{2,N}$. Respecting these facts, we can approximate TSR invariant envelopes. Digital processing of binary 2D image consists of five steps:

1. approximation of $\mathbb{B}^*_{2,N}$ as $\mathbb{B}^+_{2,N}$
2. approximation of $\mathrm{F}_2(\omega_1, \omega_2)$ as $\mathrm{F}_{2,N}(\omega_1, \omega_2)$
3. approximation of $\Phi_\mathrm{f}(\omega_1, \omega_2)$ as $\Phi^+_\mathrm{f}(\omega_1, \omega_2)$
4. approximation of $\mathrm{L_f}(\omega), \mathrm{U_f}(\omega)$ as $\mathrm{L}^+_\mathrm{f}(\omega), \mathrm{U}^+_\mathrm{f}(\omega)$
5. approximation of $\mathrm{L}^*_\mathrm{f}(\omega), \mathrm{U}^*_\mathrm{f}(\omega)$ as $\mathrm{L}^\oplus_\mathrm{f}(\omega), \mathrm{U}^\oplus_\mathrm{f}(\omega)$

The first step is only hypothetic discretization of given binary 2D image. Practical digital processing begins from $\mathbb{B}^+_{2,N}$. The second step is realizable via discrete Fourier transform (DFT), which is fast (FFT) in case when N is power of two. The third step produces a sparse matrix of normalized absolute values of DFT spectrum. The values of $\Phi^+_\mathrm{f}(\omega_1, \omega_2)$ are calculated via bicubic spline interpolation for any $\omega_1, \omega_2 \in \mathbb{R}$. It enables to calculate values of $\mathrm{L}^+_\mathrm{f}(\omega), \mathrm{U}^+_\mathrm{f}(\omega)$ for $\omega \in \mathbb{R}^+_0$ in the fourth step. The polar transform $\omega_1 = \omega\cos\varphi, \omega_2 = \omega\sin\varphi$ is used for $\varphi \in [0, 2\pi)$ and equidistant evaluation of $\Phi^+_\mathrm{f}(\omega\cos\varphi, \omega\sin\varphi)$, which helps to localize extreme values of $\mathrm{L}^+_\mathrm{f}(\omega), \mathrm{U}^+_\mathrm{f}(\omega)$. The last step analyzes the lower envelope $\mathrm{L}^+_\mathrm{f}(\omega)$ to obtain referential point $\omega^+ \in \mathbb{R}^+$ satisfying $\mathrm{L}^+_\mathrm{f}(\omega^+) = 0.5$. The adequate relativized envelopes are calculated as

$$L^\oplus_\mathrm{f}(\omega) = L^+_\mathrm{f}(\omega^+\,\omega), \quad \mathrm{U}^\oplus_\mathrm{f}(\omega) = U^+_\mathrm{f}(\omega^+\,\omega). \tag{8}$$

Decision making based on $\mathrm{L}^\oplus_\mathrm{f}(\omega), \mathrm{U}^\oplus_\mathrm{f}(\omega)$ can be realized as artificial neural network (ANN) or in the style of variation calculus.

5 Variation approach

Bipolar perceptron and its learning has been build on discrete samples of upper envelope $\mathrm{U}_{\mathrm{f}}^{*}(\omega)$ according to the formula

$$y = \operatorname{sign}\left(w_0 + \sum_{k=1}^{N} w_k\, \mathrm{U}_{\mathrm{f}}^{*}(\omega_k)\right). \tag{9}$$

In the case of equidistant values with $\triangle\omega_k \to 0_+$ and $N \to \infty$ the sum comes to integral

$$\mathrm{I}_{\mathrm{f}} = \int_a^b w(\omega)\, \mathrm{U}_{\mathrm{f}}^{*}(\omega)\, \mathrm{d}\omega, \tag{10}$$

where $w : \mathbb{R}_0^+ \to \mathbb{R}$ is unknown weight function. Denoting $w_0 = \theta$ we obtained integral perception formula

$$y = \operatorname{sign}\left(\theta + \int_a^b w(\omega)\, \mathrm{U}_{\mathrm{f}}^{*}(\omega)\, \mathrm{d}\omega\right), \tag{11}$$

which uses the whole envelope $\mathrm{U}_{\mathrm{f}}^{*}(\omega)$ instead of its samples. Let $m \in \mathbb{N}$ be number of patterns, where the whole envelope pattern is a pair $(\mathrm{e}_k(\omega), y_k^*)$, $y_k^* \in \{-1; +1\}$ is given output of k^{th} pattern and $\mathrm{e}_k : \mathbb{R}_0^+ \to [0; 1]$ is envelope $\mathrm{U}_{\mathrm{f}_k}^{*}$ of k^{th} pattern.

Here $y_k^* = +1$ means that k^{th} pattern belongs to given class and $y_k^* = -1$ means that it is false. In direct analogy with bipolar perceptron learning the learning conditions are

$$\theta + \int_a^b w(\omega)\, \mathrm{e}_k(\omega)\, \mathrm{d}\omega = y_k^* \quad \text{for } k = 1, \ldots, m. \tag{12}$$

Except of system of dependent envelopes $\mathrm{e}_1, \ldots, \mathrm{e}_m$, there are infinite number of weight functions $w(\omega)$ which solve the linear system. According to bipolar perceptron methodology the solution $w(\omega)$ must have the minimum possible Euclidean norm $\|w(\omega)\|$ or $\frac{1}{2}\|w(\omega)\|^2$ respectively. The learning task is then converted to variation optimization problem

$$\frac{1}{2}\int_a^b w^2(\omega)\, \mathrm{d}\omega \quad = \quad \min_{w,\theta} \tag{13}$$

$$\int_a^b w(\omega)\mathrm{e}_k(\omega)\, \mathrm{d}\omega \quad = \quad y_k^* - \theta \quad \text{for } k = 1, \ldots, m. \tag{14}$$

Applying the technique of Lagrange multipliers we obtain single functional

$$\mathrm{H} = \int_a^b \left(\frac{1}{2}w^2(\omega) - \sum_{k=1}^{m} \lambda_k w(\omega)\mathrm{e}_k(\omega)\right) \mathrm{d}\omega = \int_a^b \mathrm{F}(\omega, w(\omega))\, \mathrm{d}\omega. \tag{15}$$

The absence of $w'(\omega)$ in the functional H implies $\frac{\partial \mathrm{F}}{\partial w'} = 0$ and thus the Euler equation is degenerated to $\frac{\partial \mathrm{F}}{\partial w} = 0$. So, we have

$$w(\omega) - \sum_{k=1}^{m} \lambda_k \mathrm{e}_k(\omega) = 0, \tag{16}$$

which is equivalent to

$$w(\omega) = \sum_{k=1}^{m} \lambda_k \mathrm{e}_k(\omega). \tag{17}$$

The optimum weight function is only a linear combination of pattern envelopes. In the case of linear independent pattern envelopes the space of optimum weight functions has full dimension m, which enables to solve the linear system

$$\int_a^b w(\omega)\mathrm{e}_l(\omega)\,\mathrm{d}\omega = y_l^* - \theta \quad \text{for } l = 1, \ldots, m. \tag{18}$$

Then the substitution comes to

$$\sum_{k=1}^{m} \lambda_k \int_a^b \mathrm{e}_k(\omega)\mathrm{e}_l(\omega)\,\mathrm{d}\omega = y_l^* - \theta. \tag{19}$$

Denoting

$$a_{k,l} = \int_a^b \mathrm{e}_k(\omega)\mathrm{e}_l(\omega)\,\mathrm{d}\omega, \quad b_l = y_l^* - \theta, \tag{20}$$

we have $\mathsf{A}\overline{\lambda} = \overline{b}$.

The general solution is then $\overline{\lambda} = \mathsf{A}^+\overline{b}$, where A^+ is pseudoinversion of A and vector $\overline{b}$ lineary depends on threshold θ according to formula $\overline{b} = \overline{y}^* - \theta\overline{v}$ with $\overline{v} = (1, \ldots, 1)' \in \mathbb{R}^m$. The threshold value θ is also subject of optimization. The symmetry of A, A^+ matrices will help to minimize

$$\mathrm{q}(\theta) = \frac{1}{2}\int_a^b w^2(\omega)\,\mathrm{d}\omega \quad \text{for } \theta \in \mathbb{R}. \tag{21}$$

It is easy to obtain

$$\begin{aligned}
\mathrm{q}(\theta) &= \frac{1}{2}\int_a^b \left(\sum_{k=1}^{m}\sum_{l=1}^{m} \lambda_k \mathrm{e}_k(\omega)\lambda_l \mathrm{e}_l(\omega)\right)\mathrm{d}\omega = \frac{1}{2}\sum_{k=1}^{m}\sum_{l=1}^{m}\lambda_k\lambda_l \int_a^b \mathrm{e}_k(\omega)\mathrm{e}_l(\omega)\,\mathrm{d}\omega = \\
&= \frac{1}{2}\sum_{k=1}^{m}\sum_{l=1}^{m}\lambda_k\lambda_l a_{k,l} = \frac{1}{2}\overline{\lambda}'\mathsf{A}\overline{\lambda} = \frac{1}{2}(\mathsf{A}^+\overline{b})'\mathsf{A}\mathsf{A}^+\overline{b} = \frac{1}{2}\overline{b}'\mathsf{A}^+\mathsf{A}\mathsf{A}^+\overline{b} = \\
&= \frac{1}{2}\overline{b}'\mathsf{A}^+\overline{b} = \frac{1}{2}(\overline{y}^{*\prime} - \theta\,\overline{v}')\mathsf{A}^+(\overline{y}^* - \theta\,\overline{v}) = \frac{1}{2}\overline{y}^{*\prime}\mathsf{A}^+\overline{y}^* - \theta\,\overline{v}'\mathsf{A}^+\overline{y}^* + \frac{\theta^2}{2}\overline{v}'\mathsf{A}^+\overline{v}.
\end{aligned} \tag{22}$$

The optimality condition $\mathrm{q}'(\theta) = 0$ comes to threshold value

$$\theta_{opt} = \frac{\overline{v}'\mathsf{A}^+\,\overline{y}^*}{\overline{v}'\mathsf{A}^+\,\overline{v}}. \tag{23}$$

In the special case of complete envelope analysis we use $a = 0, b \to +\infty$.

The learning algorithm can be summarized as:

1. Input m, a, b as pattern number, lower and upper bound, respectively.
2. Input $\mathrm{e}_k(\omega), y_k^*$ as pattern for $k = 1, \ldots, m$.
3. Set $\overline{v} = (1, \ldots, 1)' \in \mathbb{R}^m$.
4. Form $\mathsf{A} \in \mathbb{R}^{m\times m}$ as $\mathsf{A} = \int_a^b \overline{\mathrm{e}}(\omega)\overline{\mathrm{e}}'(\omega)\,\mathrm{d}\omega$.
5. Calculate $\theta_{opt} = \frac{\overline{v}'\mathsf{A}^+\,\overline{y}^*}{\overline{v}'\mathsf{A}^+\,\overline{v}}$.
6. Form $\overline{b} \in \mathbb{R}^m$ as $\overline{b} = \overline{y}^* - \theta_{opt}\overline{v}$.

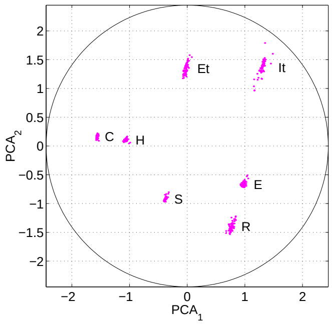

Figure 1: PCA of sampled envelopes

7. Form $\overline{\lambda} \in \mathbb{R}^m$ as $\overline{\lambda} = \mathsf{A}^{+}\overline{b}$.
8. Then $w_{opt}(\omega) = \overline{\lambda}'\overline{\mathrm{e}}(\omega)$.

The recognition scheme can be written as

$$y = \operatorname{sign}\left(\theta_{opt} + \int_a^b w_{opt}(\omega)\mathrm{e}(\omega)\,\mathrm{d}\omega\right). \tag{24}$$

6 Results

It was thought about following classes: (S) Squares, (R) Rectangles, (Et) Equilateral triangles, (It) Isosceles right triangles, (H) Hexagons, (C) Circles, (E) Elipses. Each class is represented by 70 discrete realizations. Images were tansformed and the contours of amplitude spectra and relativized envelopes were studied. Using finite number of samples from lower and upper relativized envelopes L_f^* and U_f^*, we can use PCA for patterns depicting. Two most significant principal components are able to separate seven classes. Results of PCA are depicted in Fig. 1. The TSR invariant system is able to recognize all seven classes of objects. Seven weight functions were obtained, one for each class. The first example was binary image of square. Amplitude spectrum (contours) is dpicted in Fig. 2. Resulting TSR invariant envelopes are included in Fig. 3. Acquired weight function for recognition of this class is depicted in Fig. 4. Second, binary image of circle was taken. Amplitude spectrum is dpicted in Fig. 5. Resulting TSR invariant envelopes are included in Fig. 6. Acquired weight function for recognition of this class is depicted in Fig. 7.

7 Conclusions

The article present the application of TSR invariant envelopes based on Fourier transform of 2D binary image. The envelopes were used for 2D binary image recognition. There are two basic approaches: envelope sampling in isolated points and variation approach on compact interval. Both approaches were used for classification to seven classes of binary images. Two most significant principal components of PCA are able to separate the classes in the space of sampled envelopes. The calculus of variations helps to build up a new generation of binary image recognizers, where any class is represented via weight function.

Acknowledgments

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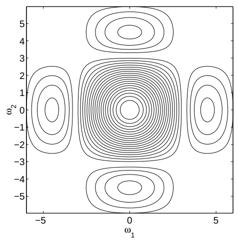

Figure 2: Spectrum of square

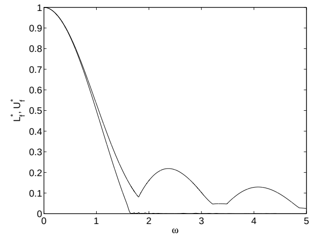

Figure 3: Envelopes of square

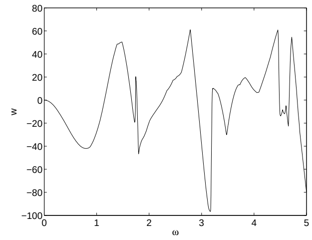

Figure 4: Weight function of square

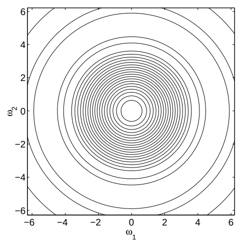

Figure 5: Spectrum of circle

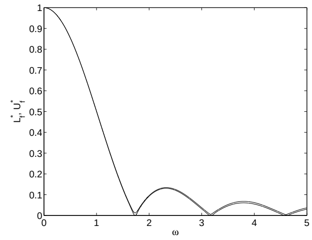

Figure 6: Envelopes of circle

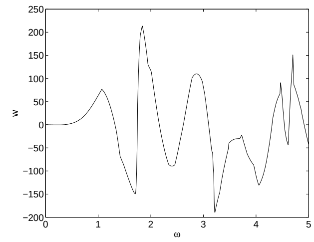

Figure 7: Weight function of circle

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