FORECASTS GENERATING FOR ARCH-GARCH PROCESSES USING THE MATLAB PROCEDURES

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Abstract

The purpose of the paper is to demonstrate the overall forecasting problems by developing and assessing models of ARCH processes forecast. Procedures were developed to determine appropriate forecasts for variance and values of the SAX index time series. These procedures were based on extension of recent developments in time series analysis and applied to forecasting systems. The comparison of classical forecasting model versus Brown´s quadratic exponential smoothing is also presented.

Keywords: *Variance function, ARCH/GARCH processes, white noise, Brown´s exponential smoothing.*

1 Introduction

The autoregressive conditional heteroscedastic (ARCH) processes described by Engle [3] are a powerful class of time series models for modelling a wide variety of financial processes. The general ARCH process in terms of ψ_t (the information set available at time *t*) can be written as

$$\begin{aligned} & y_t | \psi_{t-1} \sim \mathrm{N}(x_t^{'} b, h_t) \\ & h_t = h(\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots, \varepsilon_{t-p}, x_t, x_{t-1}, \ldots, x_{t-p}, \mathbf{a}) \\ & \varepsilon_t = y_t - \mathbf{x}_t^{'} \mathbf{b} \end{aligned} \quad (1)$$

where h_t is the variance function, h is a convenient function specification usually linear in the parameters. $\mathbf{x}_t$ is a vector of lagged endogenous and lagged and current exogenous variables, ε_t is white noise. These *x´s* and ε_t also enter the information set. Then the h_t in (1) becomes simple form $h_t = h(\psi_{t-1}, \mathbf{a})$, where $\mathbf{a}$ is a vector of unknown parameters of variance function, $\mathbf{b}$ is a vector of unknown parameters of regression model and the superscript $'$ denoting the matrix or vector transpose.

Estimating the $\mathbf{a}$ and $\mathbf{b}$ parameters can be executed by the maximum likelihood method. Inclusion of the regression $y_t = \mathbf{x}_t^{'}\mathbf{b} + \mathbf{u}_t$ and the variance function $h_t = h(\psi_{t-1}, \mathbf{a})$ leads to least number of parameters in the spirit of the parsimony (that is in simplest form).

The variance function h_t in (1) may be used to obtain easily variance forecast. In section 2, we will consider the extension of these idea and discuss the techniques for forecast variance modelling in ARCH-GARCH processes through the use of variance function expressed in terms of the $\{\varepsilon_{t-p}\}$ and $\{y_{t-p}\}$, where $p \geq 1$. In [1] several assumptions above parameters estimation of the ARCH-GARCH models were discussed. Estimation of the model parameters requires that the successive observations are represented by a linear combination of independent errors (white noise process). Test of hypotheses and interval estimation assume that the errors are normally distributed. The analyst should always consider the validity of these assumptions. In Section 2 we give some methodological remarks to the construction of point forecast for variance. In section 3 we concentrate on the modelling and forecasting ARCH-GARCH using MATLAB toolboxes: GARCH, Statistics and Optimization. To compare the forecast accuracy of the ARCH-GARCH methodology, in Section 4 this method is compared against Brown´s quadratic exponential smoothing method. We illustrate an example of these methodologies for the non-linear dependence of the stock price index SAX time series. Finally concluding remarks are presented.

2 Point forecast for variance

Once the variance function of the ARCH process has been selected, it can be used to generate forecasts for future time periods that are optimal in a minimum mean square error sense. The variance function expressed in form (1) is called the general form of the variance function. As we mentioned above, it can include error components $\{\varepsilon_t\}$, lagged dependent $\{y_{t-p}\}$, exogenous variables, and can also combine all these variables.

Denote the current period by *T*, τ periods into the future. Let $\hat{h}_{T+\tau}(T)$ represent the variance point forecast for period $T+\tau$ made at origin *T*. The forecast is generated by taking expectation at origin *T* of the variance function at time $T+\tau$. Generally, the forecast for period $T+\tau$ must be built up successively from the forecasts for periods *T+1, T+2*, ..., $T+\tau$ *-1* [4]. We suppose that in this procedure the h_{T+j} that have not occurred at time *T* are replaced by the forecasts $\hat{h}_{T+j}(T)$, the errors component ε_{T+j} that have not occurred at time *T* are replaced by zero, and the ε_{T-j} that have occurred are replaced by residuals $e_1(T-j) = h_{T-j} - \hat{h}_{T-j}(T-j-1)$. In starting the forecasting process it will be necessary to assume that $\varepsilon_{T-j} = 0$ for $T\text{-}j \le 0$.

As an illustration, consider forecasting the variance function in (1). At $T+\tau$ the function is

$$h_{T+\tau} = a_0 + a_1 y_{T+\tau-1}^2 + a_2 y_{T+\tau-2}^2 + \varepsilon_{T+\tau} \tag{2}$$

The point estimate of this function for $\tau = 1$ and taking expectation at time T on both sides of the equation, we obtain

$$E[h_{T+1}] \equiv \hat{h}_{T+1}(T) = \hat{a}_0 + \hat{a}_1 y_T^2 + \hat{a}_2 y_{T-1}^2 + e_1(T) \tag{3}$$

When we now consider the variance forecasts using the function expressed only in term of the ε variables (random shock of the model [4]), we may write

$$\begin{aligned} h_{T+\tau} &= a_1\varepsilon_{T+\tau-1} + a_2\varepsilon_{T+\tau-2} + \ \dots\ + a_{\tau-1}\varepsilon_{T+1} + \\ &\quad + a_\tau\varepsilon_T + a_{\tau+1}\varepsilon_{T-1} + \ \dots\ + \varepsilon_{T+\tau} \end{aligned} \tag{4}$$

Thus the forecasting function corresponding to Eq. (4) by taking expectation as before is

$$\hat{h}_{T+\tau}(T) = a_\tau e_1(T) + a_{\tau+1} e_1(T-1) + \ \dots \tag{5}$$

The Eq. (5) was obtained formally from (4) in which at time $t > T$ the corresponding ε_t was replaced by zero and at time $t \le T$ by replacing ε_t by $e_1(t)$. The last equation provides a simple algorithm for updating the forecasts at end of each time period. This algorithm is described in [4]. In addition, Eq. (4) and (5) admit a simple algorithm for obtaining prediction limits on the τ-step ahead variance forecast. For more details on this algorithm see [4].

3 Data and identification of ARCH-GARCH

To illustrate the ARCH-GARCH methodology, consider the stock price SAX index time readings. We would like to develop a time series model for this process so that a predictor for the process output can be developed. The data was collected for the period July 2, 1995 to October 31, 2005 which provided a total of 2510 observations (see Fig. 1).

After some experimentation, using Matlab toolboxes we have identified resulting GARCH(1,1) model

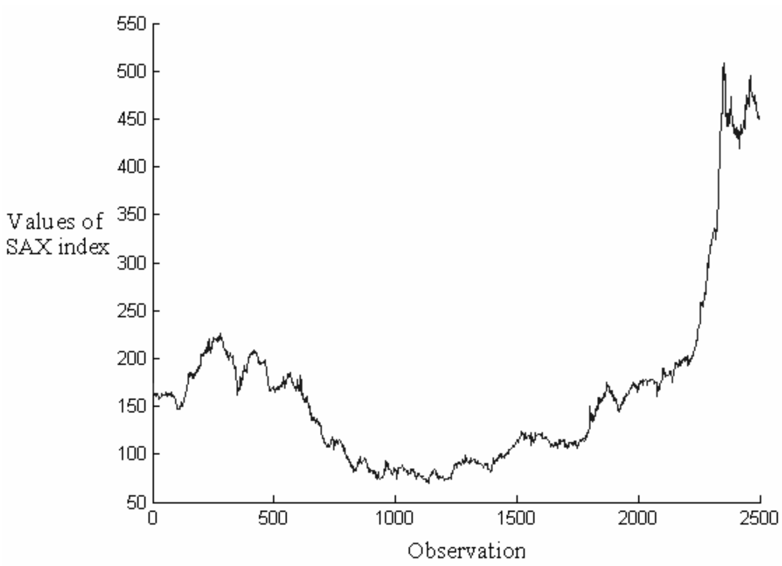

Figure. 1: The data for SAX index (July 1995-October 2005)

$$
\begin{aligned}
y_t &= -0.0001619 + \varepsilon_t \\
h_t &= 7.1803\ 10^{-6} + 0.89624 h_{t-1} + 0.068056 \varepsilon_{t-1}^2
\end{aligned}
\tag{6}
$$

The sample autocorrelation function [2] of this model for normalised residuals gives evidence that the residuals are those of a white noise process. Actual and estimated values for SAX index are graphically depicted in Fig. 2.

Next, the model for SAX time series has been developed by using only last 500 observations. To build a forecast model the sample period for analysis $y_{20č9}$, ..., y_{2450} was defined, i.e. the period over which the forecasting model was developed and the ex post forecast period (validation data set), y_{2451}, ..., y_{2510} as the time period from the first observation after the end of the sample period to the most recent observation. By using only the actual and forecast values within the ex post forecasting period only, the accuracy of the model can be calculated. The appropriate model was identified as ARMA(1,1)/GARCH(1,1) as follows

$$
\begin{aligned}
y_t &= -2.0957e - 005 + 0.93151 y_{t-1} - 0.86721 \varepsilon_{t-1} \\
h_t &= 2.4086\ 10^{-5} + 0.69207 h_{t-1} + 0.11898 \varepsilon_{t-1}^2
\end{aligned}
\tag{7}
$$

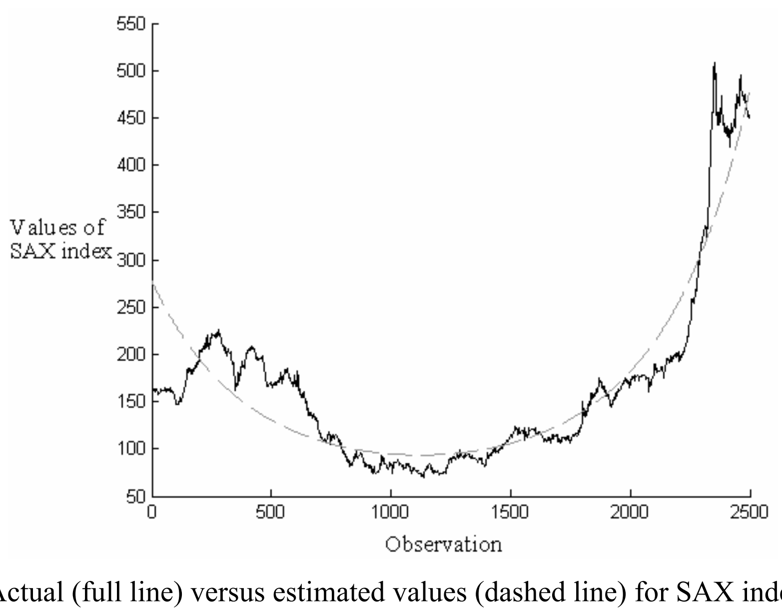

Figure 2: Actual (full line) versus estimated values (dashed line) for SAX index (GARCH(1, 1) model

The data for SAX stock price index and the ex post forecast values of the ARMA(1,1)/GARCH(1,1) estimated by model (7) and ex ante forecast for ten period are graphically depicted in Fig. 3.

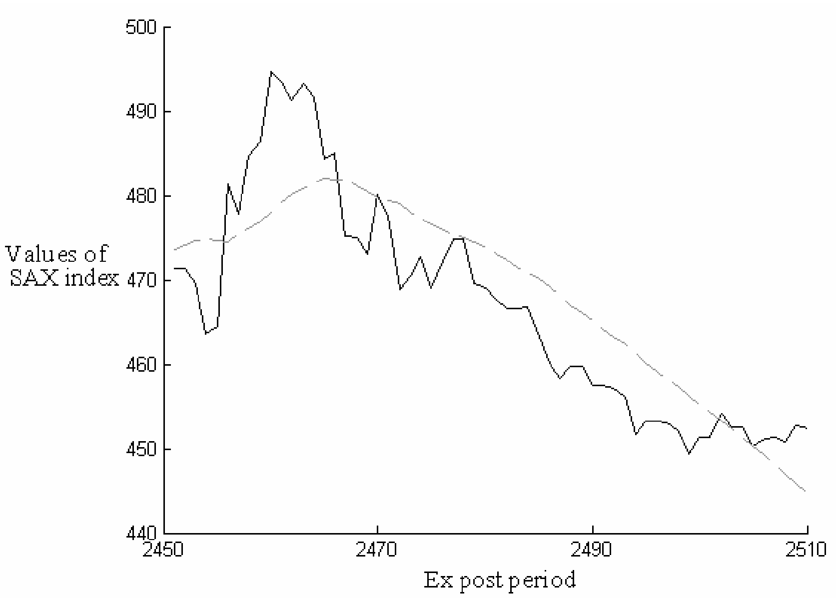

Figure: 3 Ex post forecasts and ex ante forecast for ten periods of SAX index (dashed line) and actual data

4 Brown´s quadratic exponential smoothing

To compare the predictive power of another time series models using daily data of SAX, in this section we focus on the Brown´s quadratic exponential smoothing approach. Finally we compare the forecasting accuracy of our time series models.

The data for SAX stock price index and the ex post forecast values of the Brown´s quadratic exponential smoothing approach are graphically depicted in Fig. 4.

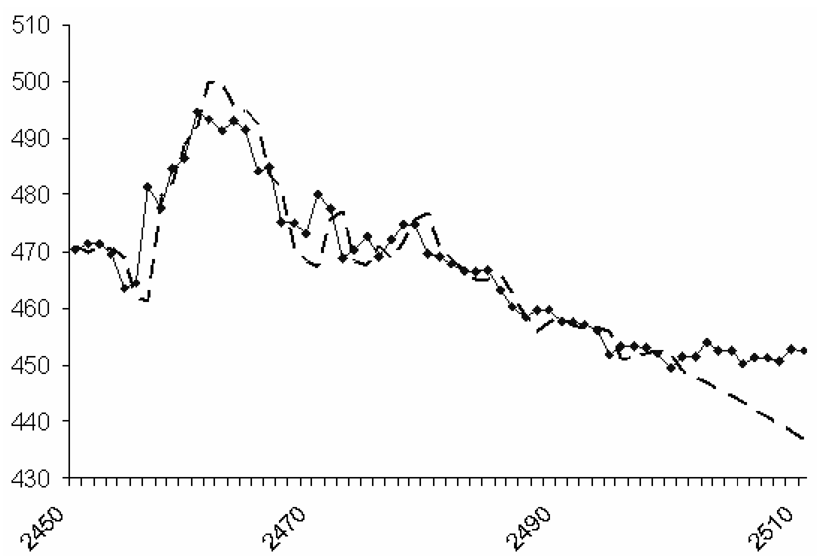

Figure 4 Brown´s quadratic exponential smoothing approach - ex post forecast and ex ante forecast for ten periods of SAX index (dashed line) and actual data

As is standard in the economic literature, we then computed the Root Mean Squared Error (RMSE). The accuracy of our ten day forecast are presented in table 1 From this table can be seen that the ARMA(1,1)/GARCH(1,1) model is the best.

Table 1: RMSE FOR TEN EX ANTE FORECASTS*

MODEL	GARCH(1, 1)	ARMA(1,1)/ GARCH(1,1)	Brown´s exp. smoothing
RMSE	37.345	3.8144	10.04

5 Conclusion

This paper has focused on the problems associated with forecasting economic variables, which disturbances follow an ARCH process. The suggested methods are applicable to process models, where the underlying forecast variance may change over time. We attempted to use the Brown´s quadratic exponential smoothing approach for forecasting an ARCH process. The use of the Brown´s quadratic exponential smoothing approach for forecasting an ARCH model of the SAX index time series had limited success. Because the results were based on chosen SAX index values and data set, they were difficult to generalise to other situations. Yet, the results certainly provide a rational way for improvement of forecasting ability in chaotic economic systems.

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