FUZZY TIME SERIES MODELLING BY SCL LEARNING

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Abstract

Based on the works [11], [22] a fuzzy time series model is proposed and applied to predict chaotic financial process. The general methodological framework of classical and fuzzy modelling of economic time series is considered. A complete fuzzy time series modelling approach is proposed. To generate fuzzy rules from data, the neural network with Supervised Competitive Learning (SCL)-based product-space clustering is used.

1 Introduction

Much of the literature in the field of the fuzzy logic and technology is focused on dynamic processes modelling with linguistic values as its observations (see e.g. [14], [15], [22]). Such a dynamic process is called fuzzy time series. This type of dynamic processes play very important role in making practical applications. Economic and statistical time series analysis is concerned with estimation of relationships among groups of variables, each of which is observed at a number of consecutive points in time. The relationships among these variables may be complicated. In particular, the value of each variable may depend on the values taken by many others in several previous time periods. Very often it is difficult to express exactly these dependencies, or there is not known hypothesis for that. Very frequently, in such cases more sofisticated approaches are considered. These approaches are based on the human experience knowledge and consist of series linguistic expressions each of which takes the form of an ´ if ... then ...´ fuzzy rule, and they are well known under the common name fuzzy controllers. But also, an expert is usually unable linguistically describe the behaviour of economic processes in particular situations. Hence, most recent researches in the fuzzy controllers design for deriving of linguistically interpreted fuzzy rules have been centered on developing automatic methods to build these fuzzy rules using a set of numerical input-output data. For applying of these methods it is supposed that a database describing previous input-output behaviour of a system is available [14]. Majority of these models and data-driven techniques rely on the use Takagi-Sugeno type controllers and fuzzy/non-fuzzy neural networks [6], [8-10], [19], [24], clustering/fuzzy-clustering and genetic algorithm approaches [3], [4], [7], [11], [12], [23], [25].

The goal of this paper is to illustrate that two distinct areas, i.e. fuzzy sets theory and computational networks, may be used to economic time series modelling. We show how to use and how to incorporate both fuzzy sets theory and computational networks to determine the fuzzy relational equations. As an application of proposed method, the estimate of the inflation is carried out in this paper. The characterisation of time series is introduced in Section 2. Quantitative modelling methods of time series are presented in Section 3 and 4. They introduce conventional and fuzzy time series modelling and show how to combine neural and fuzzy system to produce fuzzy rules. Concluding remarks are offered in Section 5.

2 Conventional and fuzzy time series

Time series models are based on the analysis of chronological sequence of observations on particular variable. Typically, in conventional time series analysis, we assume that the generating mechanism is probabilistic and that the observed values $\{x_1, x_2, ..., x_t, ...\}$ are realisations of stochastic processes $\{X_1, X_2, ..., X_t, ...\}$.

In contrast to the conventional time series, the observations of fuzzy time series are fuzzy sets (the observations of conventional time series are real numbers). Song and Chisson [22] give a thorough treatment of these models. They define a fuzzy time series as follows. Let Z_t, (t = ..., 1, 2, ...), a subset of $\Re$, be the universe of discourse on which fuzzy sets x_t^i, (i = 1, 2, ...) are defined and

X_t is the collection of x_t^i, (i = 1, 2, ...). Then X_t, (t = ..., 1, 2, ...) is called a fuzzy time series on Z_t, (t = ..., 1, 2, ...)**.**Once the variance function of the ARCH process has been selected, it can be used to generate forecasts for future time periods that are optimal in a minimum mean square error sense. The variance function expressed in form (1) is called the general form of the variance function. As we mentioned above, it can include error components {ε_t}, lagged dependent {y_{t-p}}, exogenous variables, and can also combine all these variables.

3 Quantitative time series modelling methods

In practice, there are many time series in which successive observations are dependent. This dependence can be treated here as an observational relation

$$R_o = \{(y_{t-1}, y_t), (y_{t-2}, y_{t-1}), ...\} \subseteq Y_{t-1} \times Y_t, \quad (1)$$

where Y_t, Y_{t-1} denote the variables and y_t, y_{t-1}, ... denote the observed values of Y_t and Y_{t-1} respectively.

In most real economic processes it is assumed that there exists a functional structure between Y_{t-1} and Y_t, i.e.

$$f: \ Y_{t-1} \rightarrow Y_t \quad (2)$$

belonging to a prespecified class of mappings [5]. In practice many real models of this functional structure are represented by linear relation

$$y_t = f(y_{t-1}, \phi_1, \varepsilon_t) = \phi_1 \, y_{t-1} + \varepsilon_t \quad (3)$$

where ϕ_1 is the parameter of this linear relation, ε_t is a random error or noise component that is drawn from a stable probability distribution with zero mean and constant variance.

To determine the model (3) statistical methods are used such that function (3) satisfies some optimality criterion in fitting the observed data R_o.

In the case of the fuzzy time series, the fuzzy relational equations can be employed as the models. Analogously to the conventional time series models, it is assumed that the observation at the time t accumulates the information of the observation at the previous times, i.e. there exists a fuzzy relation such that [22]

$$y_t^j = y_{t-1}^i \circ R_{ij}(t, t-1), \quad (4)$$

where $y_t^j \in Y_t$, $y_{t-1}^i \in Y_{t-1}$, $i \in I$, $j \in J$, I and J are indices sets for Y_t and Y_{t-1} respectively, "$\circ$" is the sign for the *max-min* composition, $R_{ij}(t, t-1)$ is the fuzzy relation among the observations at t and t -1 times. Then Y_t is said to be caused by Y_{t-1} only, i.e.

$$y_{t-1}^i \rightarrow y_t^j \quad (5)$$

or equivalently

$$Y_t \rightarrow Y_{t-1} \quad (6)$$

and

$$Y_t = Y_{t-1} \circ R(t, t-1), \quad (7)$$

where $R(t, t-1)$ denotes the overall relation between Y_t and Y_{t-1}. In the fuzzy relational equation (7) the overall relation $R(t, t-1)$ is calculated as the union of fuzzy relations $R_{ij}(t, t-1)$,i.e. $R(t, t-1) = \bigcup_{i,j} R_{ij}(t, t-1)$, where "$\bigcup$" is the union operator. In the following, we will use Mamdani´s method [13] to determine these relations. For simplicity, in the following discussion, we can also express y_{t-1}^i

and y_t^j as the values of membership functions for fuzzy sets y_{t-1}^i and y_t^j respectively. Since the Eq. (4) is equivalent to the linguistic conditional statement

$$\text{"if } y_{t-1}^i \text{ then } y_t^j \text{"}, \tag{8}$$

we have $R_{ij}(t,t-1) = y_{t-1}^i \times y_t^j$, where "×" is the Cartesian product and therefore

$$R(t,t-1) = \max_{i,j} \{ \min(y_{t-1}^i, y_t^j) \}. \tag{9}$$

Referring to the above definition by Song and Chisson of the fuzzy time series, in fuzzy time series model Y_t, Y_{t-1} can be understood as linguistic variables and y_t^j, y_{t-1}^i as the possible linguistic values of Y_t, Y_{t-1} respectively.

Equation (7) is called a first-order model of the fuzzy time series of Y_t with lag p = 1. This first order model can be extended to the p-th order model. See [22] for details.

4. Determination of fuzzy relations by neural networks

All the above fuzzy time series models can be determined if in particular models the fuzzy relations are known. Since finding the exact solution of fuzzy relations is generally very difficult and in practice unrealistic, hence, more sophisticated approaches are considered very frequently.

In a fuzzy system, a powerful tool for generating fuzzy rules purely from data are neural networks. Neural networks can adaptively generate the fuzzy rules in a fuzzy system by SCL-based product-space clustering technique [11]. Next, in a numerical example, we will illustrate and show, how to obtain fuzzy rules using the fuzzy sets theory and neural networks.

Let us consider a simple example. The data set used in this example (the 514 monthly inflation rates in the U.S.). A graph oh historical values of inflation is presented in Fig. 1. To build a forecast model the sample period for analysis y_1, ..., y_{344} was defined. The following statistical model was specified

$$y_t = \xi + \phi_1 y_{t-1} + \varepsilon_t, \tag{10}$$

where the variable y_t is explaned by only on its previous values, and ε_t is a white noise disturbance term. Using Levinson-Durbin algorithm [2], [18] the model (10) is statistically fitted as

$$\hat{y}_t = -0{,}1248 y_{t-1} \tag{11}$$

At this stage, we will only give some outlines to model a fuzzy time series in a fuzzy environment. The fuzzy time series modelling procedure consists of an implementation of several steps, usually as follows:

1. Define the input-output variables and the universes of discourse.
2. Define (collect) linguistic values and fuzzy sets on the universes of discourse.
3. Define (find) fuzzy relations (fuzzy rules).
4. Apply the input to the model and compute the output.
5. Defuzzify the output of the model.

From Step 1 to Step 2, the input data are fuzzified, in Step 3, analogously to the conventional model (10), the fuzzy time series model, i.e. the fuzzy relational model is created. Steps 4, 5 are considered as an application of the model (i.e., analysis of economic structures and the forecasting). Below, we will discuss these steps and apply them to the inflation time series at a more detailed level.

Firstly, in the fuzzification block, we specified input and output variables. The input variable x_{t-1} is the lagged first difference of inflation values $\{y_t\}$ and is calculated as $x_{t-1} = y_{t-1} - y_{t-2}$, t = 3, 4, ... The output variable x_t is the first difference of inflation values $\{y_t\}$ and it is calculated as $x_t = y_t - y_{t-1}$, t = 2, 3, ... The variable ranges are as follows $-0{,}75 \le x_t, x_{t-1} \le 0{,}75$. These ranges define the universe of discourse within which the data of x_{t-1} and x_t are, and on which the fuzzy sets have to be, specified. The universes of discourse were portioned into the seven intervals.

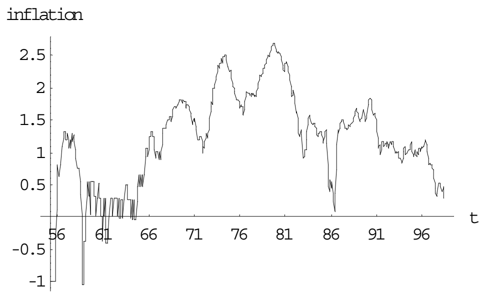

Figure 1: Natural logarithm of monthly inflation from February 1956 to November 1998.

Further, we defined cell edges with the seven intervals of the fuzzy-set values. The interval -0,75 $\leq x_t$, $x_{t-1} \leq$ 0,75 was partitioned into seven non-uniform subintervals that represented the seven fuzzy-set values. The fuzzy sets numerically represented linguistic terms. Each fuzzy variable assumed seven fuzzy-set values as follows: NL: Negative Large, NM: Negative Medium, NS: Negative Small, Z: Zero, PS: Positive Small, PM: Positive Medium, PL: Positive Large.The Cartesian product of these subsets defines 7 × 7 = 49 fuzzy cells in the input-output product space R^2. As mentioned in [10] these fuzzy cells equal fuzzy rules. Thus, there are total 49 possible rules and thus 49 possible fuzzy relations.

The neural network pictured in Fig. 2 was used to generate structured knowledge of the form „if A, then B" from a set of numerical input-output data. We can represent all possible fuzzy rules as 7-by-7 linguistic matrix (see Fig. 3). The idea is to categorise a given set or distribution of input vectors $\mathbf{x}_t = (x_{t-1}, x_t)$, t = 1, 2, ..., 344 into 7 × 7 = 49 classes, and then represent any vector just by the class into which it falls. We used SCL (Supervised Competitive Learning) [10], [14] to train the neural network in Fig. 3. The software was developed at Institute of Computer Science of Faculty of Philosophy and Science, Opava. We used 49 synaptic quantization vectors.

For each random input sample $\mathbf{x}_t = (x_{t-1}, x_t)$, the wining vector $\mathbf{w}_{i'} = (w_{1i'}, w_{2i'})$ was updated by the SCL algorithm according to

$$\left.\begin{array}{l}\widetilde{w}_{1i'} \leftarrow \widetilde{w}_{1i} + \eta\,(\widetilde{x}_{1t} - \widetilde{w}_{1i}) \\ \widetilde{w}_{2i'} \leftarrow \widetilde{w}_{2i} + \eta\,(\widetilde{x}_{2t} - \widetilde{w}_{2i})\end{array}\right\} \quad if\ i = i' \ , \qquad \left.\begin{array}{l}\widetilde{w}_{1i'} \leftarrow \widetilde{w}_{1i} - \eta\,(\widetilde{x}_{1t} - \widetilde{w}_{1i}) \\ \widetilde{w}_{2i'} \leftarrow \widetilde{w}_{2i} - \eta\,(\widetilde{x}_{2t} - \widetilde{w}_{2i})\end{array}\right\} \quad if\ i \neq i' \ ,$$

where i' is the winning unit defined $\|\widetilde{\mathbf{w}}_{i'} - \widetilde{\mathbf{x}}_t\| \leq \|\widetilde{\mathbf{w}}_i - \widetilde{\mathbf{x}}_t\|$ for all i, and where $\widetilde{\mathbf{w}}_i$ and $\widetilde{\mathbf{x}}_t$ is a normalized version of $\mathbf{w}_i$ and $\mathbf{x}_t$ respectively, η is the learning coefficient.

Supervised Competitive Learning (SCL)-based product-space clustering classified each of the 344 input-output data vectors into 9 of the 49 cells as shown in Fig. 3(a). Fig. 3(b) shows the fuzzy rule bank. We added a rule to the rule bank if the count of input-output vectors in particular cells was larger than the value 0,05N, where N = 344 is number of data pairs (x_{t-1}, x_t), t = 1, 2, ..., N in the input and output series. For example the most frequent rule represents the cell 34. From most to least important (frequent) the fuzzy rules are (PM; PS), (PS; PL), (NL; NS), (PS; PL), and (PS; PS).

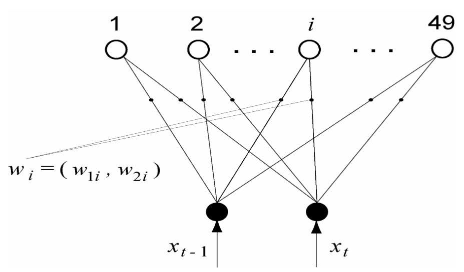

Figure 2: The topology of the network for fuzzy rules generating by SCL-based product-space clustering.

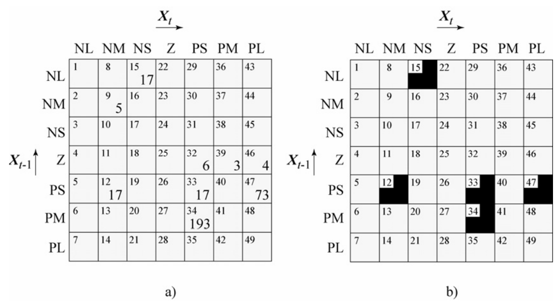

Figure 3: Distribution of input-output data (x_{t-1}, x_t) in the input-output product space $X_{t-1} \times X_t$ (a). Bank of fuzzy rules of the time series modelling system (b).

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