MHD SIMULATIONS IN PLASMA PHYSICS

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Abstract

The magnetohydrodynamic (MHD) simulations are often used in problems solved both in low- and high-temperature plasma physics. This method is very suitable for problems where the magnetic field plays important role on processes in these kind of plasmas. There is very important and interesting phenomenon called reconnection of magnetic field in high-temperature plasma physics. Explaining of this effect allow us to understand processes in this type of plasma. For example, through the reconnection of magnetic field the clouds of charged particles can leave the surface of the Sun and then implicate auroras and other effects in Earth's atmosphere. Our contribution has rather retrieval character for now. In our contribution we deal with description of MHD equations and methods of its solution, explanation of magnetic reconnection and possibilities of simulations of this phenomena and finally our next plans with Matlab package in this problem.

1 Introduction

Solar flares observed time to time in the atmosphere of the Sun are the most energetic dynamic phenomena in the whole solar system. Because of many observational evidences [1] – [5], retrieved by the *YOHKOH* and *RHESSI* satellites in X-rays, supported also by the radio data analysis [6] – [9] it is commonly accepted that the energy release in solar flares and related phenomena (e.g. solar plasma ejections, acceleration of particles, generation of shock waves, or emissions in a broad range of energies – see Fig. 1) is due to a conversion of energy stored in the magnetic field by a process known as reconnection of magnetic field lines [10].

The mechanism of magnetic reconnection can be understood most easily in 2D geometry (Fig. 2), if dissipation occurs for some reason in the current sheet, originally anti-parallel field lines can reconnect in the dissipation region and form a pair of new, highly stretched magnetic field lines. Magnetic stresses cause the strong outflow of plasma from the reconnection region, which is – because of the mass conservation – compensed by the inflow of material in the perpendicular direction. This leads to the further compression of the current sheet and thus the reconnection process continues.

In this paper we numerically study the magnetic reconnection in solar coronal plasma as the proposed main engine in flares.

2 Model

As we are interested in the simulation of reconnection on the global flare scale ($\approx 10^4$km) the most suitable description of the system plasma-magnetic field is the magneto-hydrodynamic (MHD) approach. The plasma is considered as the conducting fluid described by the set of MHD equations:

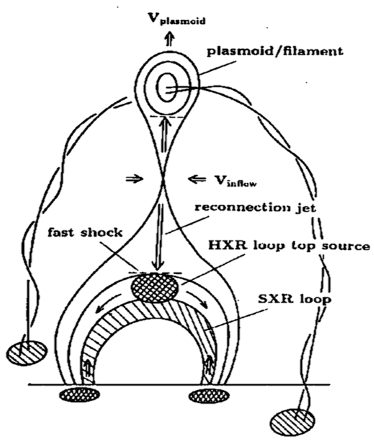

Figure 1: The scenario of solar flares based on the reconnection of magnetic field lines [3].

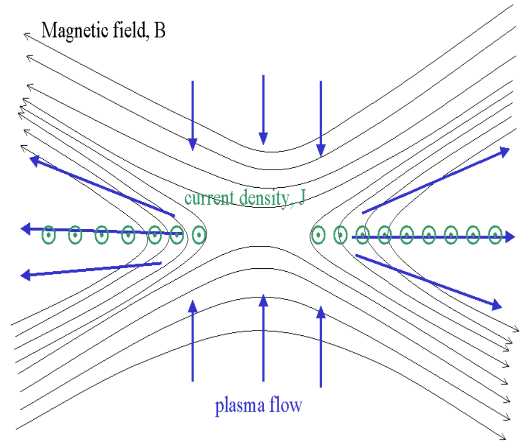

Figure 2: Schematic view of the magnetic reconnection in 2D geometry. Anti-parallel magnetic field lines (black) along the current sheet (green circles) and plasma flows are shown. The dissipation region is located in the centre of the figure.

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}\cdot(\rho \boldsymbol{v}) &= 0, \\
\rho \frac{\partial \boldsymbol{v}}{\partial t} + \rho(\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v} &= -\boldsymbol{\nabla} p + \boldsymbol{j}\times\boldsymbol{B} + \rho \boldsymbol{g}, \\
\frac{\partial \boldsymbol{B}}{\partial t} &= \boldsymbol{\nabla}\times(\boldsymbol{v}\times\boldsymbol{B}) - \boldsymbol{\nabla}\times(\eta \boldsymbol{j}), \\
\frac{\partial U}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{S} &= 0,
\end{aligned}
\tag{1}
$$

where the energy flux $\boldsymbol{S}$ and auxiliary variables (plasma pressure p and current density $\boldsymbol{j}$) are defined by the formulae [6]:

$$
\begin{aligned}
\boldsymbol{j} &= \frac{1}{\mu_0}\nabla\times\boldsymbol{B} \\
U &= \frac{p}{\gamma-1} + \frac{1}{2}\rho v^2 + \frac{B^2}{2\mu_0}, \\
\boldsymbol{S} &= \left(U + p + \frac{B^2}{2\mu_0}\right)\boldsymbol{v} - \frac{(\boldsymbol{v}\cdot\boldsymbol{B})}{\mu_0}\boldsymbol{B} + \frac{\eta}{\mu_0}\boldsymbol{j}\times\boldsymbol{B}.
\end{aligned}
$$

The solar coronal plasma can be considered as collision-less – on the other hand, the resistivity η can reach substantial value due to the plasma micro-instabilities in the highly compressed current sheet. As we are not able to resolve the dissipation scale (≈ 1m) in our global flare model, the only possible approach is the phenomenological description of physics on the sub-grid scales. Therefore, to simulate the onset of anomalous resistivity caused by the instability of electron stream forming the electric current at high electron-ion drift velocity we change the resistivity η dynamically to an anomalous value whenever the drift velocity v_{D} exceeds a given threshold v_{cr} :

$$
\eta(\boldsymbol{r},t) = \begin{cases} 0 & : \ |v_{\mathrm{D}}| \le v_{cr} \\ C\frac{(|v_{\mathrm{D}}(\boldsymbol{r},t)| - v_{cr})}{v_0} & : \ |v_{\mathrm{D}}| > v_{cr}. \end{cases}
$$

The system of equations (1) can be solved analytically only for few special cases, generally

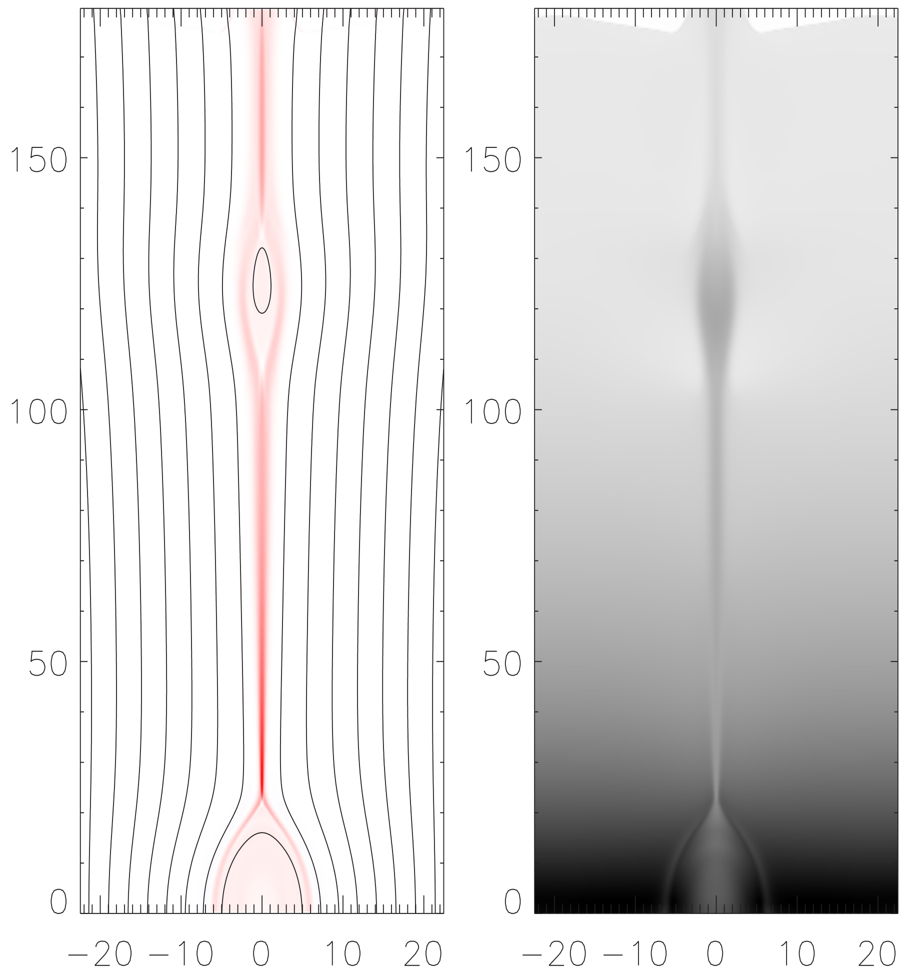

Figure 3: 2.5D MHD modelling of solar flare. Formation of raising magnetic arcade above the bottom boundary – photosphere and plasmoid ejection are clearly visible. Left panel:magnetic field lines (black contours) and the current density (red area); right panel: the density structure in gray-scale (black means higher value).

only the numerical solution is possible. For numerical integration we chose the finite-difference Lax-Wendroff scheme [11]. The used scheme, as well as most of the other modern integration schemes in (magneto)hydrodynamics, is suitable for equations in the form of conservation law:

$$\frac{\partial \boldsymbol{u}(\boldsymbol{x},t)}{\partial t} + \frac{\partial \boldsymbol{F}(\boldsymbol{u}(\boldsymbol{x},t))}{\partial \boldsymbol{x}} = 0. \tag{2}$$

Therefore the system (1) has to be rewritten into the form (2) firstly. This is fortunately possible, for example the momentum equation can be after some algebraic manipulation using the mass conservation law and the current density definition written as:

$$\frac{\partial \rho v_i}{\partial t} = -\nabla_j . \left(\rho v_i v_j - \frac{B_i B_j}{\mu_0} + \delta_{ij} (\frac{B^2}{2\mu_0} + p) \right), \tag{3}$$

The system is then described by the state vector (Eq. (2)):

$$\boldsymbol{u} = \begin{pmatrix} \rho \\ \rho v_x \\ \rho v_y \\ \rho v_z \\ B_x \\ B_y \\ B_z \\ U \end{pmatrix}, \tag{4}$$

and the divergence of corresponding flux $\boldsymbol{F}(\boldsymbol{u})$ is formed by the combination of terms on the RHS of system (1).

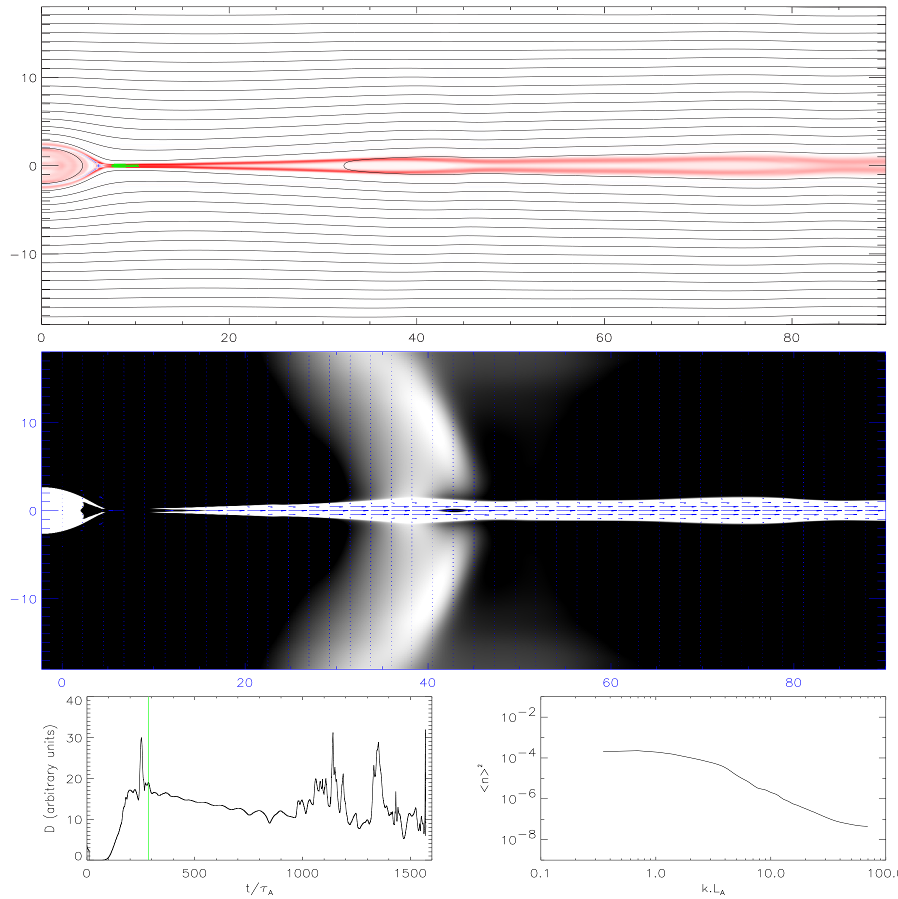

Figure 4: Shock wave generated in the dissipation region by the burst of reconnection activity (measured by the dissipated power – see the bottom-left panel) at $t \approx 250\tau_{\mathrm{A}}$. The two main panels (note their rotation in this presentation; the photosphere is in the left) show the state at $t = 285\tau_{\mathrm{A}}$; upper panel: magnetic filed lines (black), current density (red), and the dissipation area (green); middle panel: plasma density (gray-scale, white means higher value) and the velocity pattern (blue arrows). The bottom right panel show the isotropic Fourier spectrum of density variations in the out-flow jet.

The system is numerically solved in the 2.5D geometry (2D symmetry with z-component of vectors allowed) inside a rectangular box with the fixed boundary condition (simulating the solar photosphere) on the bottom and free outer boundaries. We used a vertical Harris type current sheet that was perturbed around the point $[x = 0L_{\mathrm{A}}, y = 20L_{\mathrm{A}}]$ for a finite time by anomalous resistivity as an initial state. We use dimensionless variables for convenience throughout the simulation, namely, the spatial coordinates x and y are expressed in the unit of the current sheet width L_{A} and time in Alfvén transit time $\tau_{\mathrm{A}} = L_{\mathrm{A}}/V_{\mathrm{A},0}$. $V_{\mathrm{A},0} = B_0/\sqrt{\mu_0\rho_0}$ is the asymptotic value ($x \to \pm\infty$) of the Alfvén speed at the initial state ($t = 0$). See paper [9] for details. The numerical algorithm has been implemented in C++ code, parallelised using MPI environment, and executed on the recently built *Ondrejov Cluster for Astrophysical Simulations*, see http://wave.asu.cas.cz/ocas.

3 Results

We would like to present here selected particular results of the simulation. First, we paid our attention to the global flare dynamics. The results are shown in Fig. 3. In good agreement with the supposed flare scenario Fig. 2 as well as with observations we can see the main effects of reconnection in solar flares: formation of raising arcades of magnetic loops above the photo-

sphere, conversion of magnetic energy into the plasma motion in the reconnection outflow jet, and ejection of the so called plasmoid – the dense plasma captured inside the closed O-type magnetic structure.
Further, we concentrated on another aspect of solar flares – generation of shock waves detected frequently by radio spectrographs. We have found, that the bursty character of reconnection in the long current sheets described already in [6] has a consequence – rapid energy dissipation in the resistive region results in to the formation of the MHD shock. The results are shown in Fig. 4. The shock generated by the burst of reconnection activity at $t \approx 250\tau_{\mathrm{A}}$ propagates clearly from the dissipation region. The presented snapshot of the reconnection dynamics was taken at $t = 285\tau_{\mathrm{A}}$. Kinematics of the wave front was further analysed and it was found that it is the fast magneto-sonic shock with the Mach number $M_{\mathrm{A}} = 2.56$.

4 Future plans

As we mentioned above, we have used the finite-difference Lax–Wendroff algorithm for the solution of the set of equations (1). In recent years there is often used by many authors Finite Elements Method (FEM) for the solution of equations of this type. Hence, we would like to use Matlab or COMSOL Multiphysics package for this purpose. Finally, we are going to use this software package for parallel calculations of this problem, if it is possible.

Acknowledgement

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