Symbolical Numeric Process for error transformation

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Motivation

During processing of set of a real data we had to face situation when the temporary results should be transform further via an implicit formula. The problem was how would be transform further their errors. The standard method based on function differential could not be used as it was not possible to analytically evaluated (partial) derivatives of transform function. Symbolic MATLAB toolbox cannot be used neither, as the formulas contain function which cannot be treated symbolically. Below it is described a method which solved such type of problem.

Method description

Seismic source ("origin of the earthquake") can be, under some simplification, represented by seismic moment tensor (3x3 symmetric tensor (MT), i.e. 6 independent components). MT can be further decomposed into 3 angels (named strike, dip, rake – for the definition see e.g. Aki and Richard, 1980) which describe geometrical orientation of the source and into amount of double couple (CD), compensated linear vector (CLVD) and volumetric (VOL) parts.

When it is known not only MT but also error of its components we can ask how are these errors transformed during the MT's decomposition. MT and its error were determined e.g. by Kolář 2007a, b. Using a standard approach error Δf of function $f = f(x)$ can be expressed by function's differential in form of

$$\Delta f(x^0) = \sqrt{\sum_i \left(\frac{\partial f(x^0)}{\partial x_i} \Delta x_i^0 \right)^2} \qquad ,$$

where x^0 is MT's value and Δx its error (both values are supposed to be know), function f is MT's decomposition in our case.

However MT's decomposition itself is rather complicated to evaluate partial derivatives analytically. Direct application of (MATLAB) symbolic derivative operation is also not possible as the decomposition includes mathematical operation which cannot he treated symbolically in principal (e.g. absolute value or relation operations). However, in our case we needn't to know full analytical form of function's derivatives, it is enough to know the derivatives in a particular point - in MT. For this particular case we developed a Symbolical-Numeric Process (here after SNP): we added one more dimension of size 2 to all variables in program for MT decomposition (in MATALB platform such extension can be done very easy). In the first position of an extended variable we store its symbolic value, in the second one its numerical value for particular point. Any time we need use any symbolically generally non-defined function, we look for its actual numerical value and evaluate symbolical value in concordance. Whole principle is demonstrated in following (simplified) example of modified MATLAB function for absolute value:

```
function ret=absSNP(val)
%
% evaluate abs value in SNP Method
% val=[val_symbolic, val_numeric];
%
   ret=val;
   if val(2) < 0   % numerical value is tested
       ret(1:2)=val(1:2) * -1;
   end
return
```

In such a way we modified all required functions, namely: `abs`, `min`, `max`, `atan2`, `eig`. SNP method can be suitable for errors determination for problems of similar complexity.

Method application and testing

The described method was applied on the data coming from an active experiment: fluid pumping into the KTB borehole[1]. We successfully transformed errors of MTs into errors of their decomposition. The obtained results are in a good agreement with previous numbers which were obtained by mapping of parameter space (Kolář, 2007a); graphical representation of the results is given in Fig. 1.

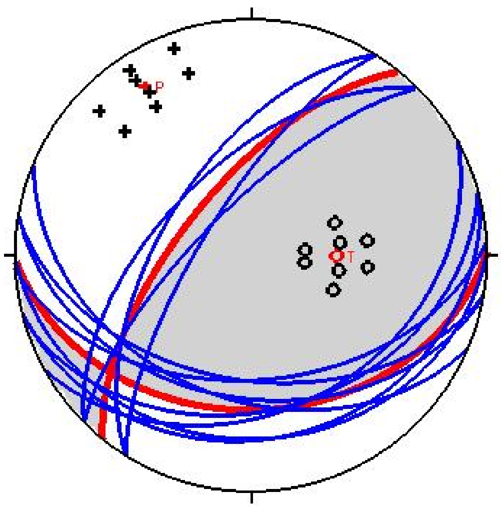

Figure 1: An example of graphical representation of a KTB event source mechanism (red thick line) and its errors (blue thin lines) - see Kolář 2007b. The figure was created by program MEPL2 – see Kolář (2007c).

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[1] KTB is a super deep (depth about 9 km) borehole situated in Germany, not far form the border with Czech Republic – for more details see e.g. Baish et al. (2002).

References

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