

APPLICATION OF ERROR MODEL TO RESOLVE CONFIGURATIONS OF SPATIAL DISTANCE INTERSECTION

Radek Dušek

University of Ostrava, Faculty of Science, Dep. of Physical Geography and Geoecology

Abstract

Surfaces of the same spatial error of an identified point are numerically resolved and displayed subsequently. The surfaces pictorially express relation between problem configuration and precision of an identified point.

1 Introduction

The error model of geodetic task addresses precision of position determination of one or more points. The model solution results in covariance matrix that will allow for design of error ellipse (for 2D tasks) or error ellipsoid (for 3D tasks) as a comprehensive precision characteristic. The error model is described e.g. in [1], [2], [3], [4] in detail.

The error model has been used to resolve configuration of spatial (3D) distance intersection. Surfaces of the same spatial error (isosurface) of an identified point are numerically resolved and displayed subsequently. The isosurfaces pictorially express relation between problem configuration and precision of an identified point.

Besides the necessity to plot isosurfaces to a plane, there is another problem with closeness of surface where „greater“ surface hide „smaller“ surface in themselves. For better transparency, surfaces are displayed in separate Figures and cross sections are used in some cases.

2 The principle of spatial distance intersection

Given:

Three points through their coordinates (A, B, C). See the Figures for coordinates of the specified points, values are in metres.

Measured:

Three distances between identified point (P) and given points – s_{PA} , s_{PB} , s_{PC} . Spatial distance intersection has been resolved for distance precision $m_s = 3$ mm.

Solvability:

If the three given points are not in line, the problem has:

- one solution if the distances intersect in the plane determined by the given points,
- two solutions if the distances intersect outside the plane. Solutions are then plane symmetric where the plane of symmetry is given by the given points,

If the three given points are lying on one line, the problem has:

- one solution, if the distances determine point on this line (at the same time, the distances lay on this line where two of the distances are sufficient to identify the point),
- Infinite number of solutions if the distances intersect outside the line. Solutions include all points of a circle given by the distances (two of the distances are sufficient to identify the circle).

Jakobi's matrix for the error model:

$$\mathbf{J}(\mathbf{x}) = \mathbf{A}_1 = \begin{pmatrix} \frac{\partial s_{PA}}{\partial X_p} & \frac{\partial s_{PA}}{\partial Y_p} & \frac{\partial s_{PA}}{\partial Z_p} \\ \frac{\partial s_{PB}}{\partial X_p} & \frac{\partial s_{PB}}{\partial Y_p} & \frac{\partial s_{PB}}{\partial Z_p} \\ \frac{\partial s_{PC}}{\partial X_p} & \frac{\partial s_{PC}}{\partial Y_p} & \frac{\partial s_{PC}}{\partial Z_p} \end{pmatrix} = \begin{pmatrix} -\frac{\Delta X_{PA}}{s_{PA}} & -\frac{\Delta Y_{PA}}{s_{PA}} & -\frac{\Delta Z_{PA}}{s_{PA}} \\ -\frac{\Delta X_{PB}}{s_{PB}} & -\frac{\Delta Y_{PB}}{s_{PB}} & -\frac{\Delta Z_{PB}}{s_{PB}} \\ -\frac{\Delta X_{PC}}{s_{PC}} & -\frac{\Delta Y_{PC}}{s_{PC}} & -\frac{\Delta Z_{PC}}{s_{PC}} \end{pmatrix}.$$

3 Numeric solution of isosurfaces

The solution took place in the Matlab program in two steps programmed in separate M-files:

1. spatial error has been calculated for a regular network of points in space (interval 20 m) using the model solution,
2. isosurfaces for specific values of the mean spatial error (m_{PR}) have been calculated and displayed subsequently using the functions of *patch*, *isosurface* and *isonormals*. The specified functions defined as simple commands in Matlab allowed for the following:
 - a. solution of a complex spatial interpolation to determine behaviour of isosurfaces,
 - b. easy subsequent display of such isosurfaces.

For examples of resulting isosurfaces see Figures 2 through 8.

4 Assessment of configuration effect

Three given points form a triangle which may have different shape; basic assessment is done for equilateral triangle. Three given points define a plane, which is the plane of symmetry of the spatial error isosurfaces; the assessment is carried out for one of symmetric parts.

The point where the distances intersects under right angles is determined most precisely (Fig. 1). The isosurfaces form the surface of a flattened solid (Fig. 2) stretching in the direction of the given points with the error increasing (Fig. 3). When the spatial error increases further, the isosurfaces converge into a spherical surface passing through the given points (Figs. 4 and 6).

If the given points form an acute-angled or obtuse-angled triangle, the isosurfaces of small values of spatial error will change according to Figures 7 and 8 (compare to Fig. 2 and 3). With increasing error values, differences in isosurfaces are smaller, they converge into spherical surface again.

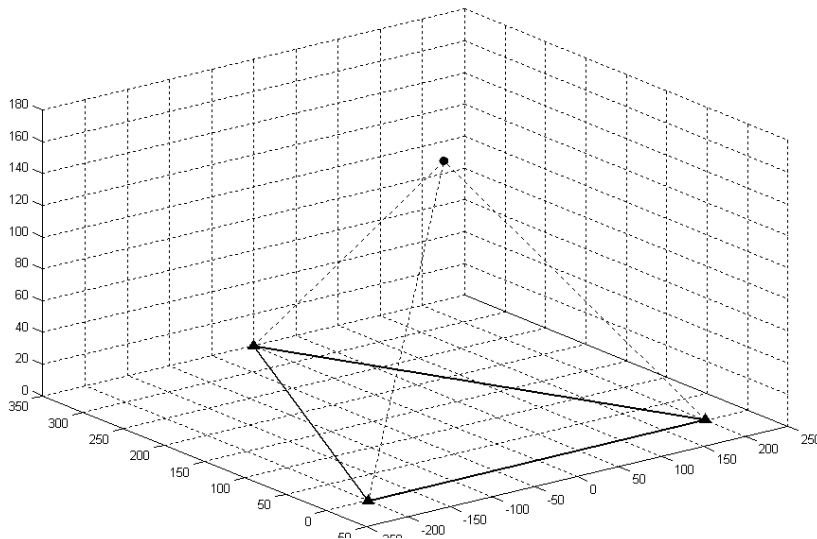


Figure 1: Optimum configuration of spatial distance intersection.

5 Conclusion

The procedure used, combining a model solution of a geodetic task and potential of the Matlab software, has shown to be very appropriate for design and visualization of isosurfaces of constant spatial error of a point to be identified.

References

- [1] R. Dušek. *Lokální geodetické sítě v hornické krajině*. HGF VŠB-TUO, Ostrava, 2005.
- [2] R. Dušek – J. Vlasák. *Geodézie 40: příklady a návody na cvičení*. ČVUT, Praha, 1998. 127 p. ISBN 80-01-01929-2.
- [3] M. Ingeduld et al. *Geodézie: metody výpočtu a vyrovnání geodetických sítí*. ČVUT, Praha, 1990. 242 p. ISBN 80-01-00333-7.
- [4] J. Jandourek. *Geodézie IV: úprava měřených veličin před výpočty, geodetická úloha a její kvalitativní hodnocení*. ČVUT, Praha, 1995. 149 p. ISBN 80-01-01330-8.

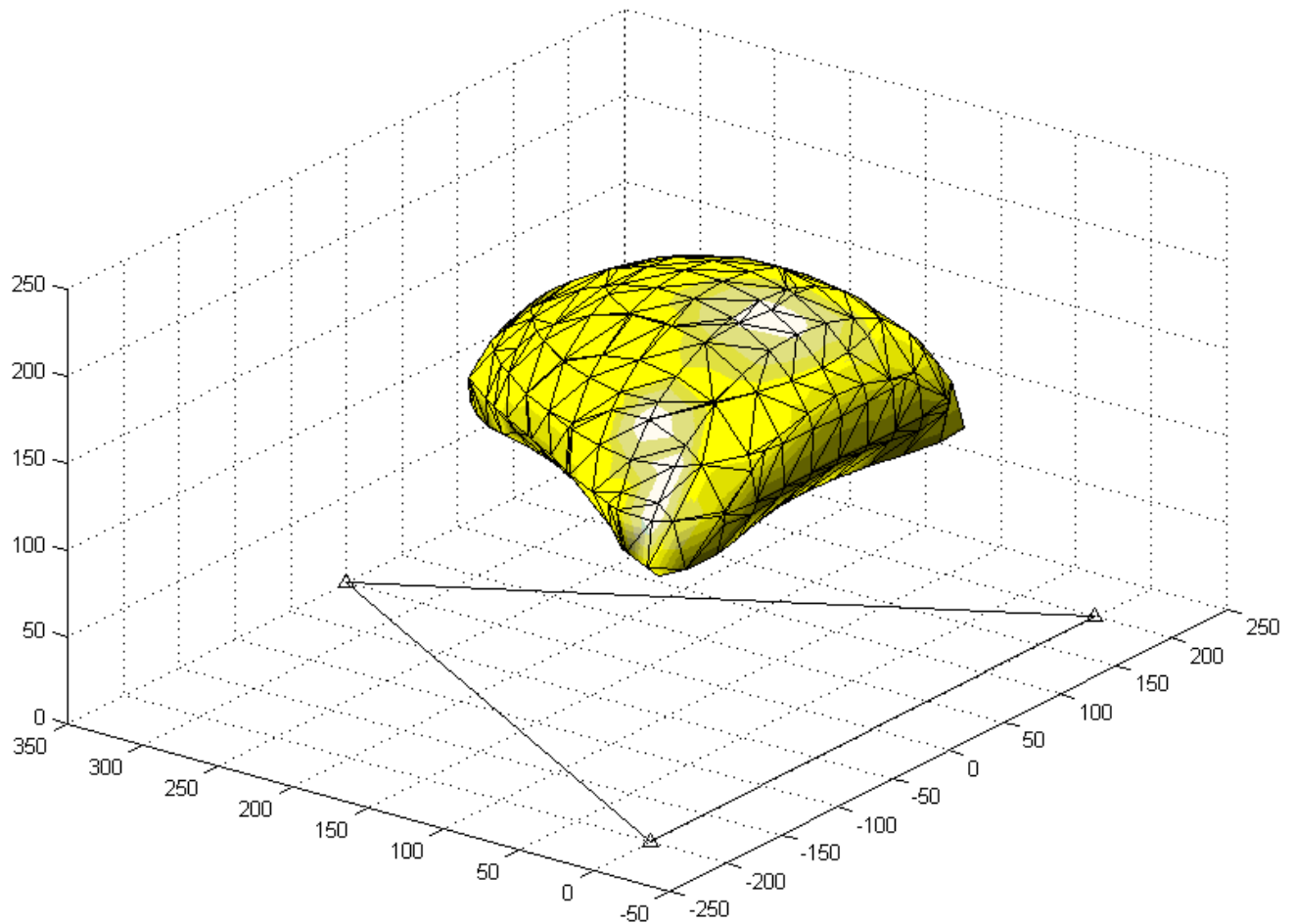


Figure 2: Isosurface for $m_{PR} = 5.5$ mm, given points form an equilateral triangle.

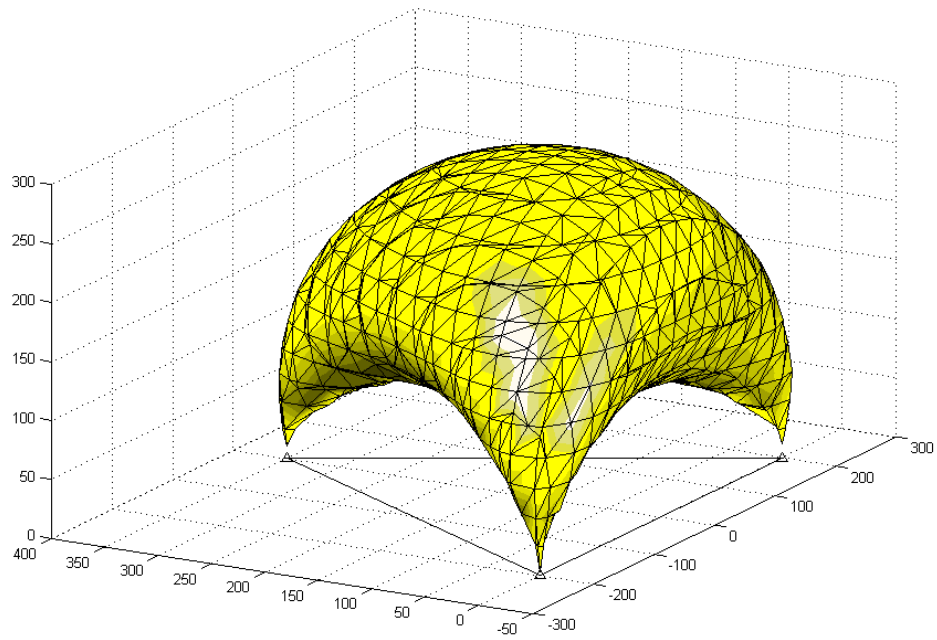


Figure 3: Isosurface for $m_{PR} = 6.0$ mm, given points form an equilateral triangle.

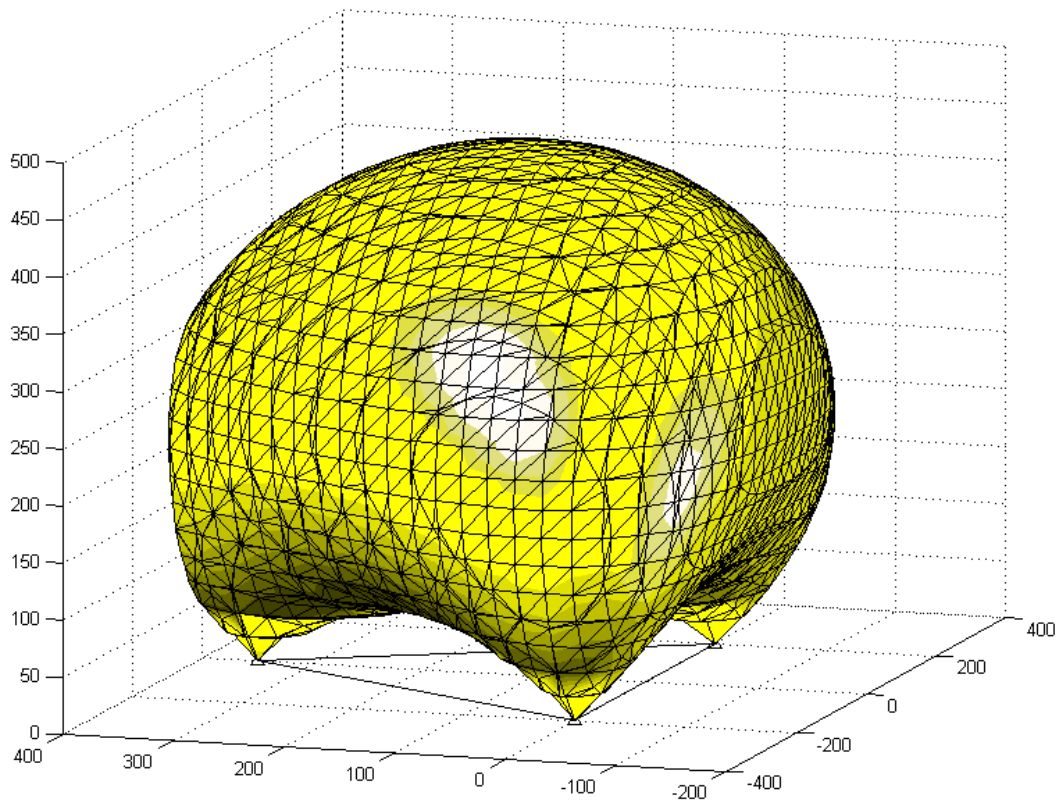


Figure 4: Isosurface for $m_{PR} = 8.0$ mm, given points form an equilateral triangle.

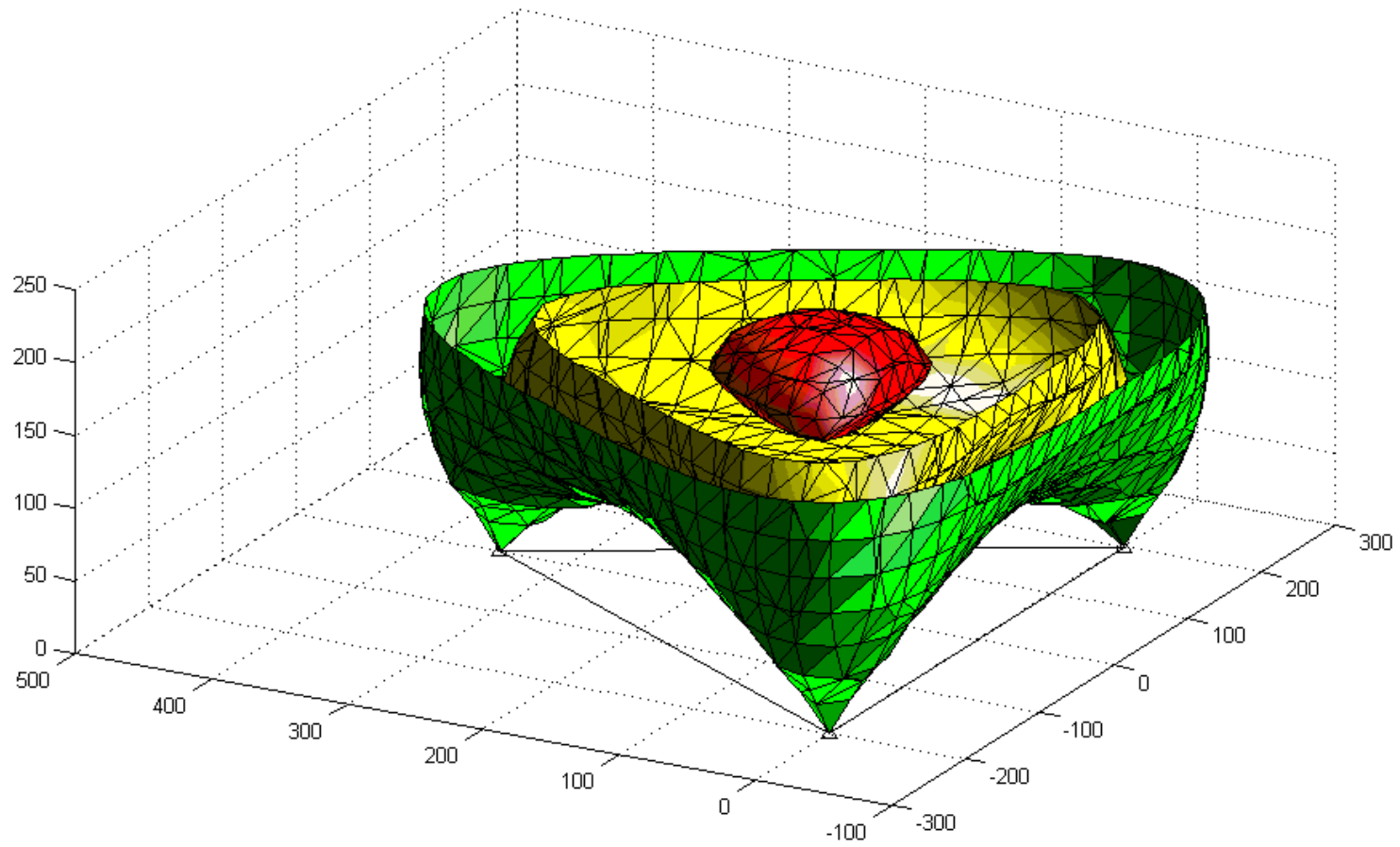


Figure 5: Section through isosurfaces for m_{PR} 5.3 mm, 6.0 mm, 7.0 mm, given points form an equilateral triangle.

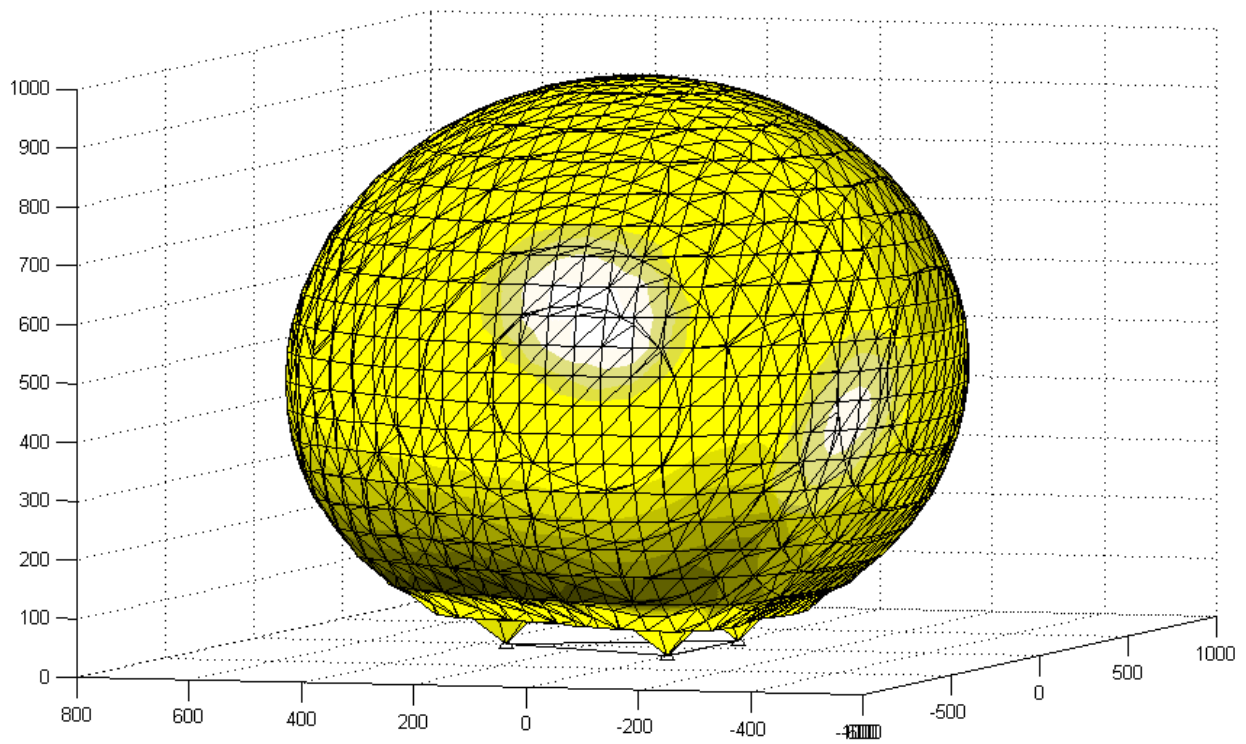


Figure 6: Isosurface for $m_{PR} = 15.0$ mm, given points form an equilateral triangle.

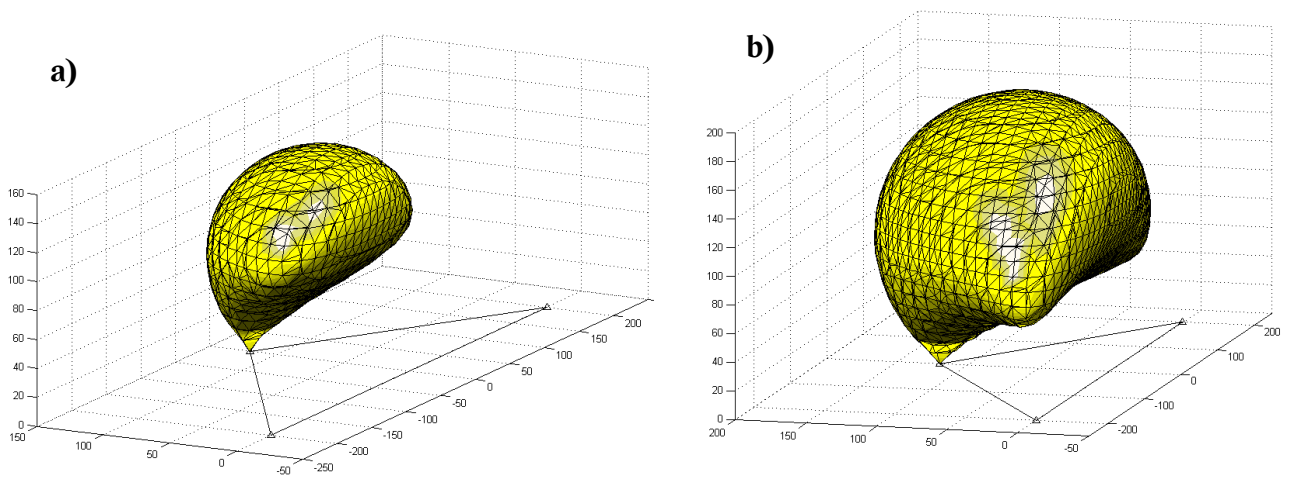


Figure 7: Given points form an obtuse-angled triangle, values of mean spatial error:
 a) $m_{PR} = 7.0$ mm, b) $m_{PR} = 8.0$ mm.

