

# PREDICTIVE CONTROL BY DISTRIBUTED PARAMETER SYSTEMS BLOCKSET FOR MATLAB & SIMULINK

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## Abstract

**Simulation of model predictive control of casting mold temperature field is presented. Temperature fields of the mould have been modeled and studied using a finite element method based software package, ProCAST, and numerical models in the form of lumped-input/distributed-output systems have been obtained. For model predictive control synthesis the Model Predictive Control Toolbox was used. Simulation have been realized using the Distributed Parameter Systems Blockset for MATLAB & Simulink, which is a third-party MathWorks product.**

## 1 Introduction

Most of the dynamical systems analyzed in engineering practice are distributed parameter systems (DPS) given by numerical structures on complex-shape 3D definition domains. Dynamical characteristics obtained by numerical methods offer wide possibilities for control of systems as DPS.

In this paper an enhancement of DPS Blockset, the Predictive Control Synthesis Block is presented. Predictive control synthesis is decomposed into time and space components. Space control problems are being solved by approximation methods, problems concerning control in time are based on predictive control synthesis methods for lumping systems. For software implementation, functions and methods from the Model Predictive Control Toolbox (MathWorks) are used. In this paper, as a demonstration of achieved results, predictive control of preheating of a complex-shape casting die is presented.

Temperature control in the casting die could have considerable implication for the elimination of defects that may arise in the process. For the analysis of casting process dynamics as DPS, benchmark of the casting processes was designed. It has to be able to embrace typical effects and critical moments of casting. Temperature fields of the casting mould have been modeled and studied using ProCAST, which is a FEM based software package. For control synthesis purpose, numerical models in the form of lumped-input/distributed-output systems are made up by means of finite element analysis.

## 2 DPS/LDS dynamics and control

Distributed parameter systems, whose state or output variables  $X(x,y,z,t)/Y(x,y,z,t)$  are distributed variables, or fields of variables, are in practice very frequently found in the form of lumped-input/distributed-output systems (LDS), (Hulkó, 1987, 1998) see Fig. 1.

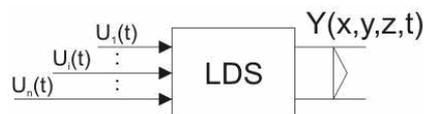


Figure 1: DPS in the form of LDS:

$\{U_i(t)\}_{i=1,n}$  - lumped input variables,  $Y(x,y,z,t)$  - distributed output variable

For this DPS/LDS representation it is possible to obtain discrete transfer functions (1) between  $i$ -th input variable and the corresponding partial distributed output variable at point  $\bar{x}_i = (x_i, y_i, z_i)$ . In space dependence there are reduced steady-state transient step responses (2):

$$\{SH_i(\bar{x}_i, z)\}_{i=1,n} \quad (1)$$

$$\{\mathcal{N}HR_i(\bar{x}, \infty)\}_{i=1,n} \quad (2)$$

Decomposition of dynamics outlined above enables decomposition of the control synthesis to time and space control problems in DPS feedback control loop, see Fig. 2.

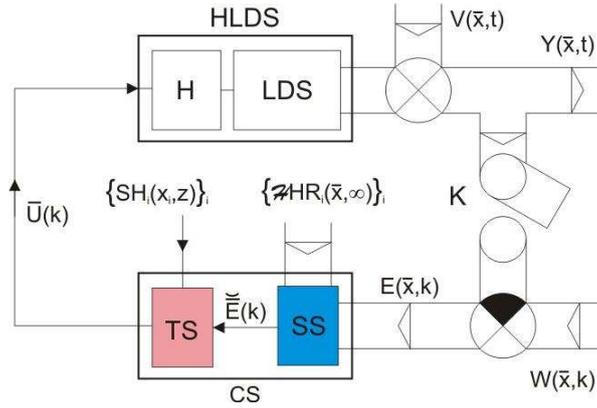


Figure 2: DPS feedback control loop:  
 HLDS – LDS with zero-order holds  $\{H_i\}_i$  on the input, CS – control synthesis,  
 TS – control synthesis in time domain, SS – control synthesis in space domain,  
 K – time/space sampling,  $Y(\bar{x}, t)$  – distributed controlled variable,  $W(\bar{x}, k)$  – control variable,  
 $V(\bar{x}, t)$  – disturbance variable,  $E(\bar{x}, k)$  – control error

In the SS block, approximation of distributed control error  $E(\bar{x}, k)$ , on the set of reduced steady-state distributed step responses  $\{ZHR_i(\bar{x}, \infty)\}_i$ , is solved.

$$\min_{E_i} \left\| E(\bar{x}, k) - \sum_{i=1}^n E_i(k) ZHR_i(\bar{x}, \infty) \right\| = \left\| E(\bar{x}, k) - \sum_{i=1}^n \tilde{E}_i(k) ZHR_i(\bar{x}, \infty) \right\| \quad (3)$$

Next, the control errors vector  $\tilde{E}(k) = \{\tilde{E}_i(k)\}_i$  enters into the block TS, where the vector components  $\{\tilde{E}_i(k)\}_i$  are fed through the inputs of controllers  $\{R_i(z)\}_i$  and the sequence of control variables  $\bar{U}(k)$  is generated. Tuning of controllers is suitable to do according dynamics given by transfer functions (1).

### 3 Basic concept of Model Predictive Control

Representation of DPS in the form of LDS enables to use various methods of control synthesis well-known in lumping systems control, including the Model Predictive Control (MPC) as well. The MPC concept for single-input single-output (SISO) systems is usually supposed according to Fig. 3.

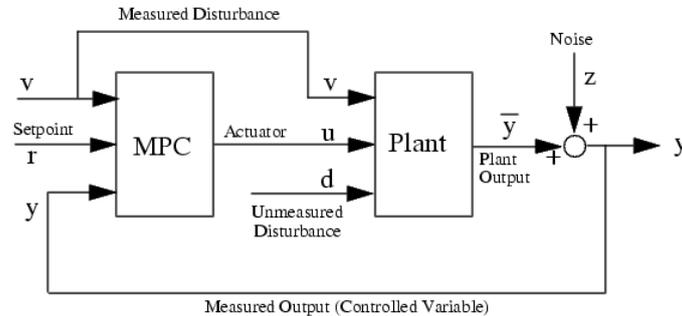


Figure 3: Block Diagram of a SISO Model Predictive Control

The main objective is to hold a single output  $\bar{y}$ , at a reference value (or setpoint)  $r$  by adjusting a single manipulated variable (or actuator)  $u$ . Model Predictive Controller is designed to achieve the control objective. It requires a plant model, which defines the mathematical relationship between the plant inputs and outputs. The controller uses it to predict plant behavior.

A Model Predictive Controller automates a target system (the plant) by combining a prediction strategy and a control strategy. An approximate linear plant model provides the prediction. The control strategy compares predicted plant signals to a set of objectives, then adjusts available actuators to achieve the objectives while respecting the plant constraints. Such constraints can include the physical limits of the actuator, boundaries of safe operation, and lower limits for product quality.

#### 4 Distributed Parameter Systems Blockset for MATLAB & Simulink

The Distributed Parameter Systems Blockset is a blockset for use with MATLAB & Simulink for distributed parameter systems and their applications in modeling, control and design of dynamical systems given on complex 3D definition domains, see Fig. 4, or web site [www.dpscontrol.sk](http://www.dpscontrol.sk).

The **HLDS** block models controlled distributed parameter systems as lumped-input/distributed-output systems with zero-order hold units. **DPS Control Synthesis** provides feedback to distributed parameter controlled systems in control loops with blocks for **PID, algebraic, state space and robust** control. The block **DPS Input** generates distributed quantities which can be used as distributed control variables or distributed disturbances, etc. **DPS Display** presents distributed quantities with many options including export to AVI files. The block **DPS Space Synthesis** performs space synthesis as an approximation problem. The block **Tutorial** presents methodological framework for formulation and solution of distributed parameter systems of control. The block **Show** contains motivation examples: Control of temperature field of 3D metal body, Control of 3D beam of „smart“ structure, Adaptive control of glass furnace and Groundwater remediation control. The block **Demos** contains examples oriented to methodology of modeling and control synthesis. The **DPS Wizard** gives a guide for arrangement and setting distributed parameter control loops in step-by-step operation, by means of five model examples on 1D-3D with default parameters,.

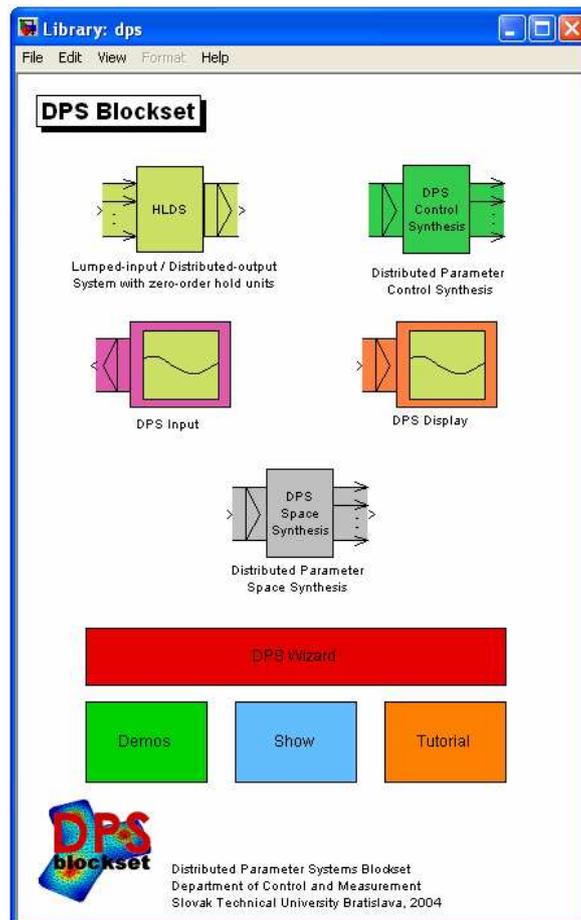


Figure 4: Library of DPS Blockset for MATLAB & Simulink

## 5 Modeling of temperature fields of the mould in casting process

Temperature field of the mould in the casting process is a typical case of DPS, where the input/output relation is useful to be modeled as LDS. The benchmark casting plant is built of the steel mold with water-cooled copper inserts (chills), electric heating elements (lumped inputs) and thermocouples, see Fig. 5.

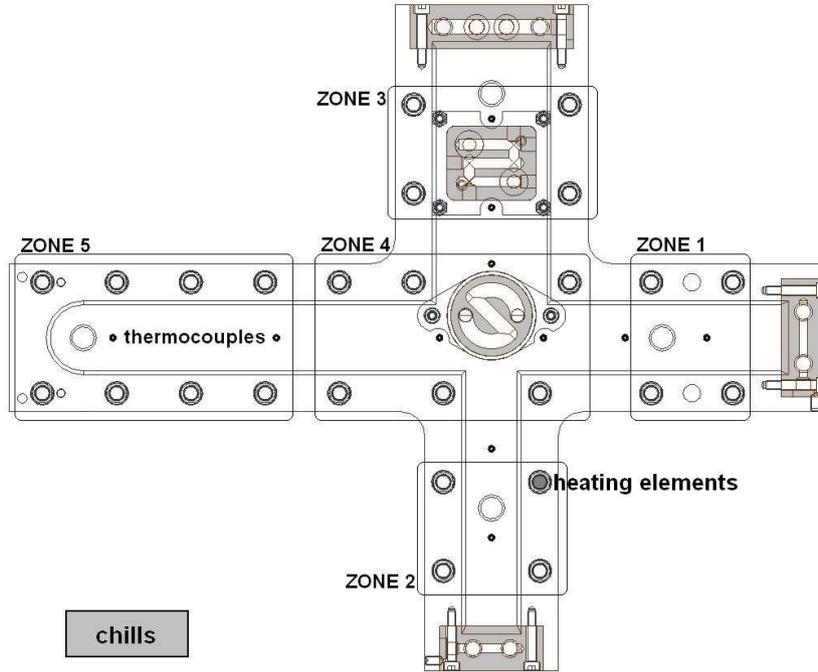


Fig. 5. Steel mold of the benchmark casting plant

Temperature distribution in the mold in the definition domain  $\Omega \in E_3$  is modeled by PDE of parabolic type in the form:

$$\frac{\partial T(\bar{x}, t)}{\partial t} - a \nabla^2 T(\bar{x}, t) = \sum_i u_i(\bar{x}, t) \quad (4)$$

where  $a = \lambda / \rho c$  is temperature conductivity ( $\text{m}^2 \cdot \text{s}^{-1}$ ).

For lumped input variables  $\{u_i\}_{i=1,5}$  in the form of step functions, which affect on sub-domains  $\{\Omega_i\}_{i=1,5}$  all necessary lumped and distributed responses were computed by FEM in the software environment ProCAST.

## 6 Simulation of Model Predictive Control of temperature field

MPC controller design procedure outlined above is now applied for the control of temperature field of the mold in the casting process.

In the MATLAB & Simulink environment by means of both, the DPS Blockset and Model Predictive Control Toolbox, distributed parameter system of MPC is set up, see Fig. 6. It is the feedback control loop, where the block MPC\_DPS includes both, time and space control synthesis, see Fig 7. In this case, the control system consists of five single parameter control loops, where MPC controllers are used, see Fig.8.

MPC synthesis controllers were adjusted in order to assure aperiodic running of the quadratic norm of distributed control error. Weights of manipulated variables, manipulated variables rate and output variables were assuming 0.0, 0.1 and 1.0. Results of the MPC control process are on Figures 10, 11.

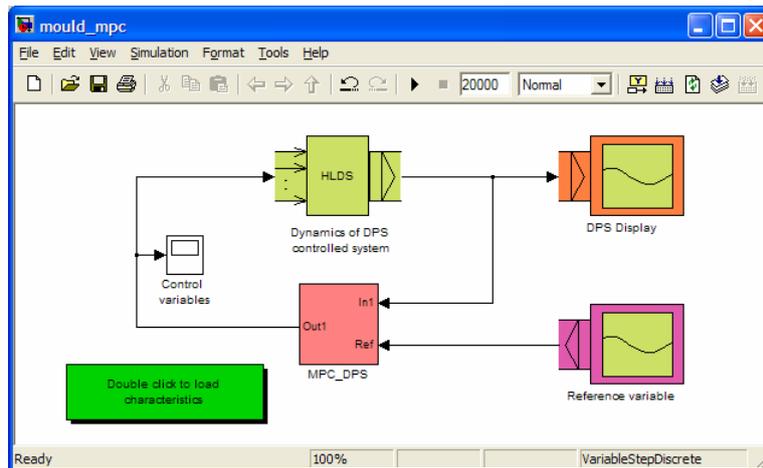


Figure 6: MPC DPS loop in DPS Blockset for MATLAB & Simulink

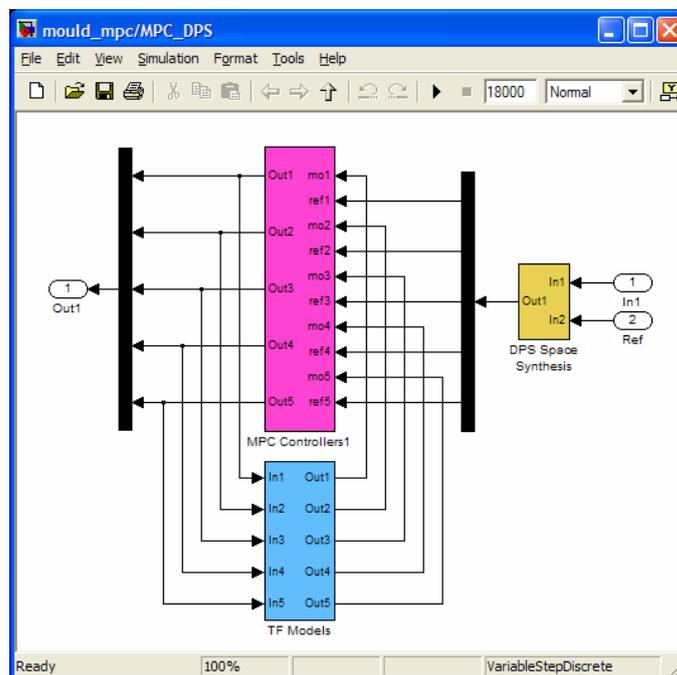


Figure 7: Internal structure of MPC\_DPS block

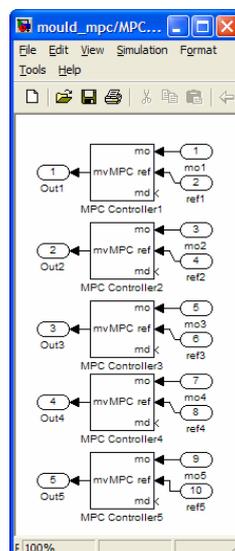


Figure 8: Block of MPC Controllers

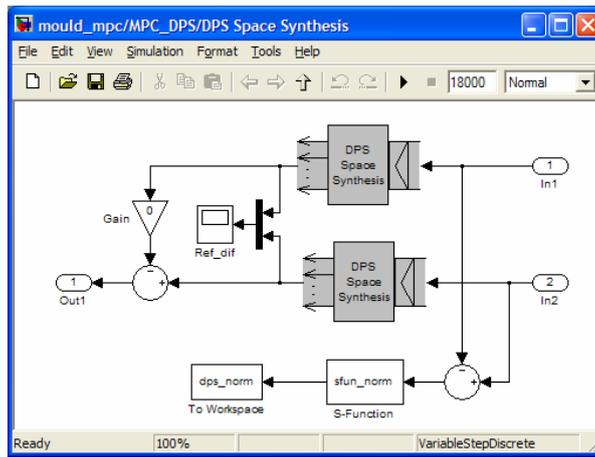


Figure 9: DPS Space synthesis

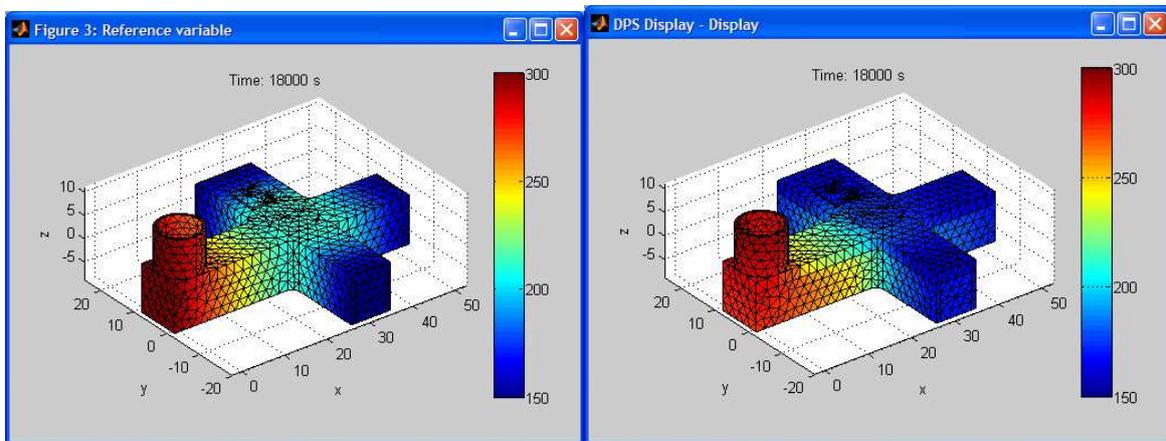


Figure 10: Distributed reference variable  $W(x,y,t)$  and controlled variable  $Y(x,y,t)$ ,  $t \rightarrow \infty$

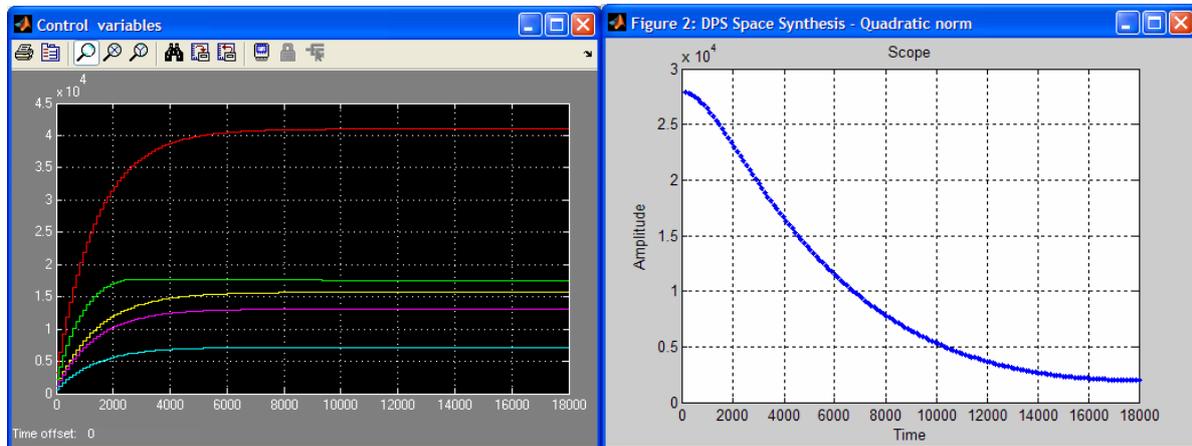


Figure 11: Control variables  $U_i(k)$  and quadratic norm of distributed control error  $\|E(k)\|$

## 7 Conclusion

Methodical approach presented in the paper demonstrates simple possibilities, how to exploit distributed dynamical characteristics, obtained by numerical FEM analysis of systems on complex definition domains for MPC controller synthesis of DPS. The DPS Blockset for MATLAB & Simulink and Model Predictive Control Toolbox provide efficient software for this kind of problems.

## ACKNOWLEDGEMENT

The work has been financially supported by the ESF project JPD 3 2005/NP1-047 „PhD students for Modern Industrial Automation in SR“, code No. 13120200115 and by the Slovak Scientific Grant Agency VEGA project „Control methods of Distributed Parameter Systems given on Complex Definition Domains with Demonstrations in MATLAB & Simulink“ (grant 1/2051/05).

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