MATLAB REALIZATION OF A NEW METHOD FOR IDENTIFICATION OF SYSTEMS OF ARBITRARY REAL ORDER

T. Skovranek[1], *V. Despotovic*[2], *I. Podlubny*[1], *I. Petras*[1], *D. Bednarova*[1]

[1] Technical University of Kosice
Faculty of BERG, Institute of Control and Informatization of Production Processes
Kosice, Slovak Republic
[2] University of Belgrade
Technical Faculty of Bor, Department of Electromechanical Engineering
Bor, Serbia

Abstract

Present paper deals with using the total least squares method (TLSM), the so called orthogonal distance regression (ODR) for system identification, instead of the classical least squares method (LSM), as ODR is a suitable tool for fitting lines and surfaces in multidimensional space, while even the use of classical regression in 2D is not so natural. A new method for identification of systems of arbitrary real order based on numerical solution of fractional differential equations and orthogonal distance fitting is proposed. In order to define a model of observed data a system of fractional-order differential equations (FODEs) will be considered, which solution represents the optimal fitting of given points in space. The solution is obtained using Grünwald-Letnikov definition of fractional derivative, which allows to get an approximate numerical solution to given analytical systems of FODEs. For obtaining of both, the sum of distances of the data points from the fitting line in 2D and 3D space and the numerical solution of a system of FODEs, MATLAB-functions were developed. The sum of orthogonal distances between the data points in space and a fitting line is considered as the criterion for optimization, which is performed using *fminsearchbnd*-function (created by John D'Errico). The results are presented in 2-dimensional and 3-dimensional space.

1 Introduction

System identification is a general term to describe mathematical tools and algorithms that build dynamical models from experimental data. Having the model of a system is often very important for analysis, simulation, control system design etc. In present paper a new method for identification of systems of arbitrary real order is proposed, which is based on numerical solution of a set of fractional differential equations. In order to successfully solve a problem of system identification, parameters of the differential equations (including orders of differentiation which are considered as arbitrary real or "fractional") have to be evaluated, and a criterion must be determined, by which the estimation process will be done optimally.

LSM was used among scientists and researchers for long period of time as an universal tool for finding best-fitting curve to a given set of points. However, there are other, more general methods, one of them TLSM [2, 3] (as well known as ODR), which advantages listed for example in [6] show the justification of using. ODR uses perpendicular (orthogonal) distances between given points and fitting curve, what is more natural than using vertical offsets, as in LSM.

Although the fractional calculus and the idea of fractional order operators is over three centuries old, the interest of scientists increased mainly in last decades [5, 4, 7]. Fractional-order differential equations (FODE) and their numerical solution is the topic of many authors (e.g. Diethelm, He, Podlubny, Lubich, Adomian, Momani, etc.). It is due to better possibilities of description of dynamical systems, as FODEs provide a powerful instrument for description of memory and hereditary properties of systems in comparison to integer-order models, where such effects are neglected.

2 Identification of linear fractional order systems using orthogonal distance fitting

The main goal of this paper is the MATLAB realization of a state space description of a dynamical system using a set of FODEs. Mathematical model of this system, is represented by a set of FODEs, which numerical solution corresponds to the optimal fitting of given points in space. Criterion of optimization is the sum of shortest distances between given points in space and fitting line. The analysis is performed in 2D and 3D space.

Let us propose a system of FODEs, written in analytical form, to which a numerical solution will be sought:

$$\begin{aligned} D^{(\alpha)}x &= a_1 \cdot x + a_2 \cdot y + a_3 \cdot z + C_1 \\ D^{(\beta)}y &= a_4 \cdot x + a_5 \cdot y + a_6 \cdot z + C_2 \\ D^{(\gamma)}z &= a_7 \cdot x + a_8 \cdot y + a_9 \cdot z + C_3. \end{aligned} \tag{1}$$

The orders of differentiation (α, β, γ) are fractional, in interval $0 < \alpha, \beta, \gamma \leq 1$.

Let us recall here the Grünwald-Letnikov definition of fractional derivative. For numerical approximation of a fractional order derivation we will use the Grünwald-Letnikov definition in the form [7]:

$${}_aD_t^\alpha f(t) = \lim_{h \to 0} \frac{{}_a\triangle_h^\alpha f(t)}{h^\alpha}, \quad {}_a\triangle_t^\alpha f(t) = \sum_{j=0}^{[\frac{t-a}{h}]} (-1)^j \binom{\alpha}{j} f(t - jh), \tag{2}$$

where $[\frac{t-a}{h}]$ means the integer part of $\frac{t-a}{h}$. From the definition (2) we can yield the formal relationship for numerical approximation of a fractional order derivation [7]:

$${}_aD_t^\alpha f(t) \approx {}_a\triangle_h^\alpha f(t). \tag{3}$$

The previous equations can be rewritten into the form [7]:

$$f_h^{(\alpha)}(t) = \frac{1}{h^\alpha} \sum_{j=0}^{n} (-1)^j \binom{\alpha}{j} f(t - jh), \tag{4}$$

where the expression $(-1)^j \binom{\alpha}{j}$ is the binomial coefficient, and can be written as follows [1]:

$$b_j^{(\alpha)} = (-1)^j \binom{\alpha}{j}, \qquad j = 0, 1, 2, \ldots. \tag{5}$$

The α in (5) represents the order of derivation. Binomial coefficients can be evaluated using recurrence relationship as in [1]:

$$b_0^{(\alpha)} = 1, \qquad b_j^{(\alpha)} = \left(1 - \frac{1+\alpha}{j}\right) b_{j-1}^{(\alpha)}, \qquad j = 1, 2, 3, \ldots. \tag{6}$$

Applying definitions (4)-(6) on (1) we obtain a numerical interpretation of the system of three linear FODEs:

$$\begin{aligned}
(a_1 - h^{-\alpha} \cdot b_0^{(\alpha)}) \cdot x_m + a_2 \cdot y_m + a_3 \cdot z_m &= h^{-\alpha} \cdot \sum b_j^{(\alpha)} \cdot x_{m-j} - C_1 \\
a_4 \cdot x_m + (a_5 - h^{-\beta} \cdot b_0^{(\beta)}) \cdot y_m + a_6 \cdot z_m &= h^{-\beta} \cdot \sum b_j^{(\beta)} \cdot y_{m-j} - C_2 \\
a_7 \cdot x_m + a_8 \cdot y_m + (a_9 - h^{-\gamma} \cdot b_0^{(\gamma)}) \cdot z_m &= h^{-\gamma} \cdot \sum b_j^{(\gamma)} \cdot z_{m-j} - C_3.
\end{aligned} \tag{7}$$

3 MATLAB realization of the data-fitting

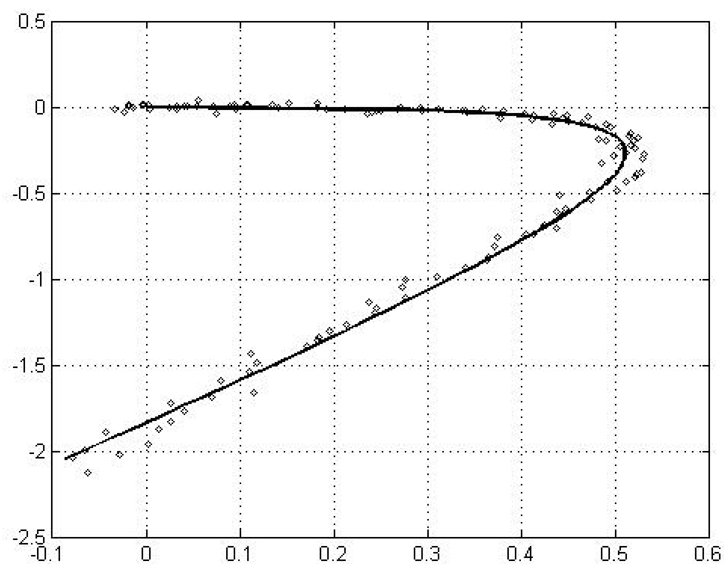


Figure 1: Fitting of data-points in 2D space

The mathematical model for finding of the best fitting curve for a given set of points, proposed in previous section, is used. In order to solve system identification problem represented by the set of equations (1) or (7), respectively, a MATLAB function for evaluating of the fractional derivatives was developed.

The coefficients $C_1, C_2, C_3,\ a_k, k = 1, 2, \ldots, 9$ and the orders of derivation α, β, γ in (7) are inputs of the optimization function *fminsearchbnd*, where the criterion of the optimization process is the sum of closest (perpendicular) distances of the data-points from the fitting line, obtained using MATLAB function *dsearchn*.

Optimization is bound constrained using *fminsearchbnd* MATLAB function (created by John D'Errico), where bounds represent limits for search of coefficients in equation (7), as well as limits for the fractional orders. The *fminsearchbnd*-function is used exactly like standard MATLAB function *fminsearch*, except that bounds are applied to the variables. The bounds

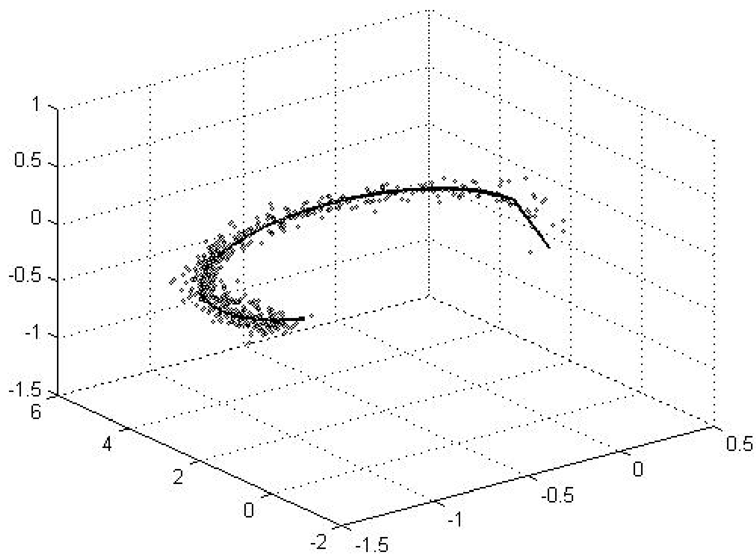


Figure 2: Fitting of data-points in 3D space

are inclusive inequalities, which admit the boundary values themselves, but will not permit any function evaluations outside the bounds.

We are now dealing with a simple system of linear algebraic equations, which can be written in the matrix form:

$$A \cdot X = B \tag{8}$$

where A is the matrix of the coefficients that corresponds to "state matrix" in state space:

$$A = \begin{bmatrix} a_1 - h^{-\alpha} \cdot b_0^{(\alpha)} & a_2 & a_3 \\ a_4 & a_5 - h^{-\beta} \cdot b_0^{(\beta)} & a_6 \\ a_7 & a_8 & a_9 - h^{-\gamma} \cdot b_0^{(\gamma)} \end{bmatrix},$$

the right side of the system is represented by B as:

$$B = \begin{bmatrix} h^{-\alpha} \cdot \sum b_j^{(\alpha)} \cdot x_{m-j} - C_1 \\ h^{-\beta} \cdot \sum b_j^{(\beta)} \cdot y_{m-j} - C_2 \\ h^{-\gamma} \cdot \sum b_j^{(\gamma)} \cdot z_{m-j} - C_3 \end{bmatrix}$$

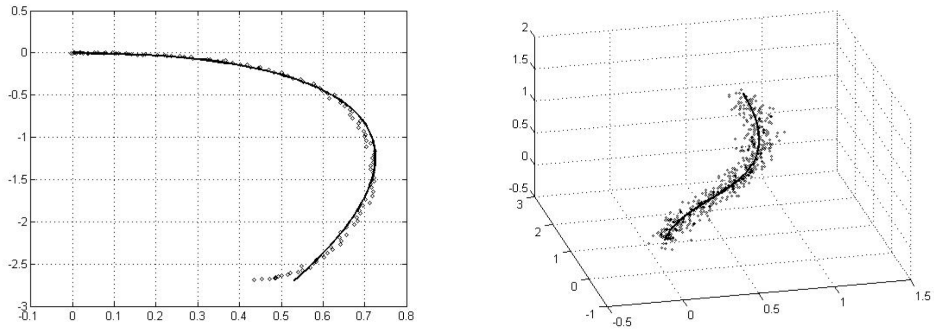
and X is the vector of solutions that corresponds to "state vector" in state space:

$$X = [x \quad y]^T, \qquad X = [x \quad y \quad z]^T.$$

Now we can easily determine X as:

$$X = A^{-1} \cdot B. \tag{9}$$

The variables of X ($[x, y]$ for 2D case and $[x, y, z]$ for 3D case) are vectors, so finally X is a matrix of coordinates of the points, which are lying on the fitting line in each discrete time step h. In Figure 1 and Figure 2 the solutions for 2D and 3D case are shown. The points represent sample experimental data, and the fitting line represents the solution of the system (7). A good match between the data points and fitting line can be observed.



(a) in 2D space

(b) in 3D space

Figure 3: Fitting of data-points where the orders of derivation are in interval $1 < \alpha, \beta, \gamma \leq 2$

4 Conclusions

System identification problem was solved by finding the best fitting line to given experimental data. The sum of distances of the data points from the fitting line was taken as the criterion for optimization process, where all the parameters of the considered system of equations (including the orders of derivations) were optimized to fit the data optimally. This method allows consideration of higher orders of derivatives in the equations (7), as shown in Figure 3. MATLAB was used for development of tools for realization of a new method for identification of systems of fractional order.

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T. Skovranek
tomas.skovranek@tuke.sk

V. Despotovic
vdespotovic@tf.bor.ac.yu

I. Podlubny
igor.podlubny@tuke.sk

I. Petras
ivo.petras@tuke.sk

D. Bednarova
dagmar.bednarova@tuke.sk