MODELLING, SIMULATION AND OPTIMIZATION OF DAMPING

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Abstract

Modelling, simulation and optimization of different systems were an integral part of the design and construction of systems. The diagram documenting all phenomena influencing behaviour of the system, arrangement a set of dynamic equations and creating a simulation numerical model are the prerequisites for creating a functional and credible simulation model. Vibration in systems has two effects. First, the very high peak accelerations can mean that the effective weight of the vibration systems increases several-fold, and this may cause its destruction. Secondly, people near the systems feel these accelerations, which can be uncomfortable or even dangerous. A vibration mass is used to protect the systems from steady-state harmonic disturbance. Depending on the driving frequency of the original system, the vibrating mass needs to be carefully tuned, that is, to choose adequate values of stiffness, damping and eventual mass, so that motion of the original mass is a minimum. The tuneable vibration absorber is advantageous primarily in that reduces the amplitude of vibrations in the systems by, for example unbalanced rotors, crank gear, clearances in bearings, oscillation of the moving driven parts, transient loading by diverging and coasting of driving motors, etc. The present article will discuss the method online optimization of the vibration and damping control for systems by using a vibration tuneable mass with damping. The analyses and Matlab m-file for the auto-tuning control have been used. The aim of the paper is to acquaint the reader with the design of the incorporated absorber to the vibration system, which makes the suppression of the vibration of the systems to a minimum possible. The resulting vertical trajectory and acceleration of the car body showed that absorption using a springs and dampers reduces the vibration amplitude of the car body. Car body vibrations on uneven highways affect car lifetime, passenger comfort, and in turn cause damage to the road. Absorption system reduces these negative influences and considerably improves the driving properties of the car. It is possible to change the parameters of the springs while driving and thus control the driving properties and comfort. Before the realization of vibration absorption in a mechanical arrangement it is important to carry out a numerical solution. The numerical solution for car body absorption, in cooperation with the Matlab-Simulink program, is shown on model of the car.

1 Model of the System

On the Fig. 1 is shown the model of the car with four degrees of freedom x, φ, x_3, x_4. The car is moving on uneven road, where displacement *s(t)= s cos Ωt.* From the road then acting on the ripe force

$$F(t) = s\sqrt{(k_2^2 + b_2^2\Omega^2)}\cos(\Omega t + \psi) = F\cos(\Omega t + \psi), \tag{1}$$

where $F = s\sqrt{(k_2^2 + b_2^2\Omega^2)}$, $\psi = \dfrac{b_2}{k_2}\Omega$.

In the time t=0 is the force $F(t)$ maximum and then displacement ψ=0. The force acting on the ripe from the road is

$$F(t) = F\cos\Omega t. \tag{2}$$

The relations among the degree of freedom x, φ, x_1, x_2 are (Fig. 1)

$$x_1 = x + l_1\varphi, \quad x_2 = x - l_2\varphi. \tag{3}$$

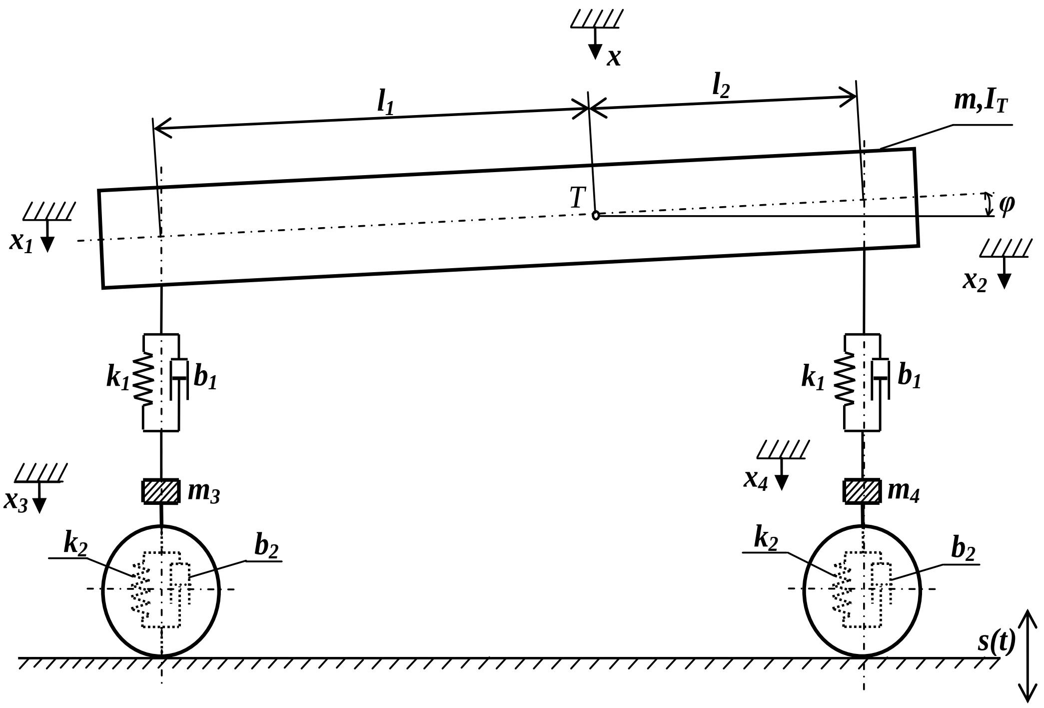

Figure 1: Model of the car with tires moving on uneven road

2 Minimize criterion

Equation of motion of this system is

$$\mathbf{m}\ddot{\bar{\mathbf{x}}} = \mathbf{k}\bar{\mathbf{x}} + \mathbf{b}\dot{\bar{\mathbf{x}}} + \mathbf{F}, \tag{4}$$

where

$$\mathbf{m} = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_T & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix}, \quad \mathbf{k} = \begin{bmatrix} -2k_1 & k_1(l_2 - l_1) & k_1 & k_1 \\ k_1(l_2 - l_1) & k_1(l_1^2 - l_2^2) & l_1k_1 & -l_2k_1 \\ k_1 & l_1k_1 & -k_1 - k_2 & 0 \\ k_1 & -l_2k_1 & 0 & -k_1 - k_2 \end{bmatrix}, \tag{5}$$

$$\mathbf{b} = \begin{bmatrix} -b_1 & -b_1 & b_1 & b_1 \\ -l_1b_1 & l_2b_1 & l_1b_1 & -l_2b_1 \\ b_1 & 0 & -b_1 - b_2 & 0 \\ 0 & b_1 & 0 & -b_1 - b_2 \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} 0 \\ 0 \\ F(t) \\ F(t) \end{bmatrix}, \quad \bar{\mathbf{x}} = \begin{bmatrix} x \\ \varphi \\ x_3 \\ x_4 \end{bmatrix}. \tag{6}$$

We introduce now

$$\dot{\bar{\mathbf{x}}} = \bar{\mathbf{x}}_1, \qquad \bar{\mathbf{x}} = \bar{\mathbf{x}}_2, \tag{7}$$

Then equation of motion (1) is

$$\begin{gathered} \mathbf{m}\dot{\bar{\mathbf{x}}}_1 = \mathbf{k}\dot{\bar{\mathbf{x}}}_1 + \mathbf{b}\bar{\mathbf{x}}_2 + \mathbf{F}, \qquad \dot{\bar{\mathbf{x}}}_2 = \bar{\mathbf{x}}_1 \quad \Rightarrow \\ \dot{\bar{\mathbf{x}}}_1 = \mathbf{m}^{-1}\mathbf{k}\dot{\bar{\mathbf{x}}}_1 + \mathbf{m}^{-1}\mathbf{b}\bar{\mathbf{x}}_2 + \mathbf{m}^{-1}\mathbf{F}, \quad \dot{\bar{\mathbf{x}}}_2 = \bar{\mathbf{x}}_1. \end{gathered} \tag{8}$$

Equations of motion (8) are also possible written in the form

$$\dot{\hat{\mathbf{x}}} = A\hat{\mathbf{x}} + BF, \quad \hat{y} = C\hat{\mathbf{x}}, \tag{9}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{m}^{-1}\mathbf{b} & \mathbf{m}^{-1}\mathbf{k} \\ \mathbf{I_T} & \mathbf{0} \end{bmatrix}, \quad \hat{\mathbf{x}} = \begin{bmatrix} \overline{\mathbf{x}}_1 \\ \overline{\mathbf{x}}_2 \end{bmatrix}, \quad \hat{\mathbf{y}} = \begin{bmatrix} x \\ \varphi \end{bmatrix},$$
$$\mathbf{B} = \begin{bmatrix} \mathbf{m} \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}. \tag{10}$$

When we introduce Laplace transformation we obtain

$$p\hat{\mathbf{x}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{BF}, \quad \hat{\mathbf{y}} = \mathbf{C}\hat{\mathbf{x}}. \tag{11}$$

The transfer function **G**(p) then from equations (11) is

$$\hat{\mathbf{y}} = \mathbf{C}(p\mathbf{I_T} - \mathbf{A})^{-1}\mathbf{BF},$$

Where the transfer function has form

$$\mathbf{G}(p) = \mathbf{C}(p\mathbf{I_T} - \mathbf{A})^{-1}. \tag{12}$$

When optimizing the response of an existing system it is conventional to minimize the maximum response. The minimize criterion is

$$\min_{k_1, b_1} \max |G(p)| \tag{13}$$

The Simulink scheme of the solution and optimization is shown on the Fig. 2.

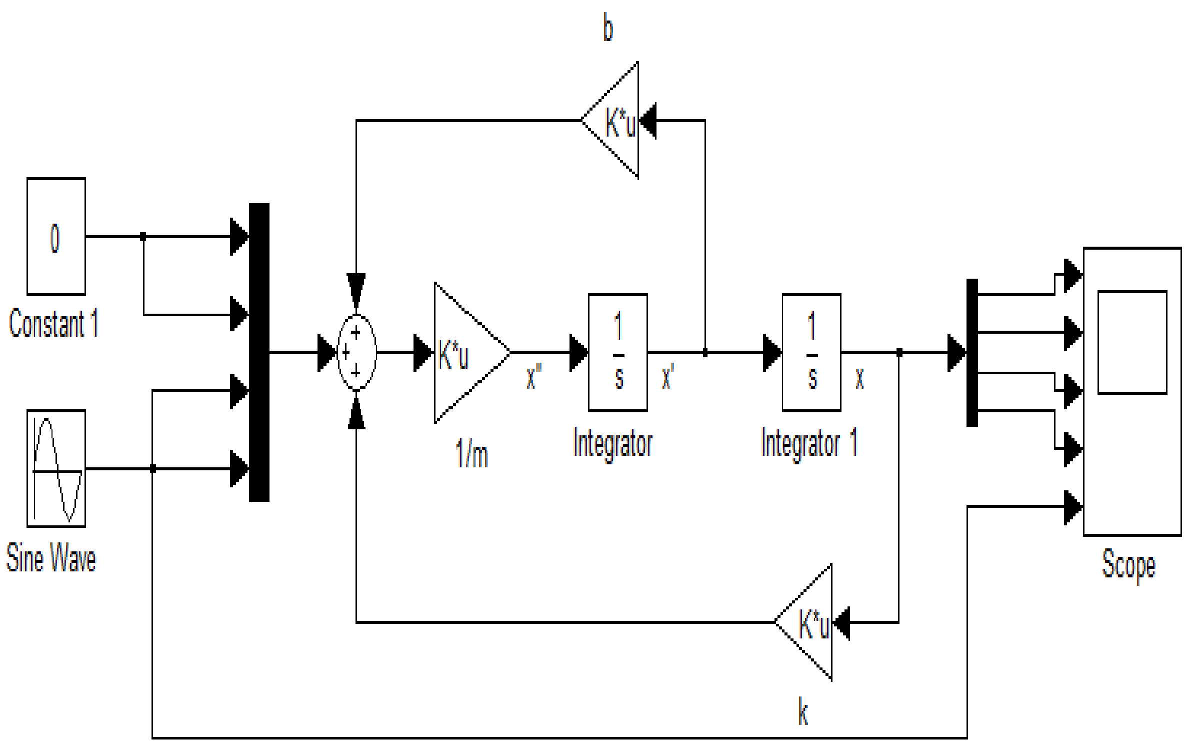

Figure 2 :Simulink scheme of the damping of the car

3 Results and conclusion

On the Fig. 3 is shown solution of the minimize criterion (13) of the transfer function (12).

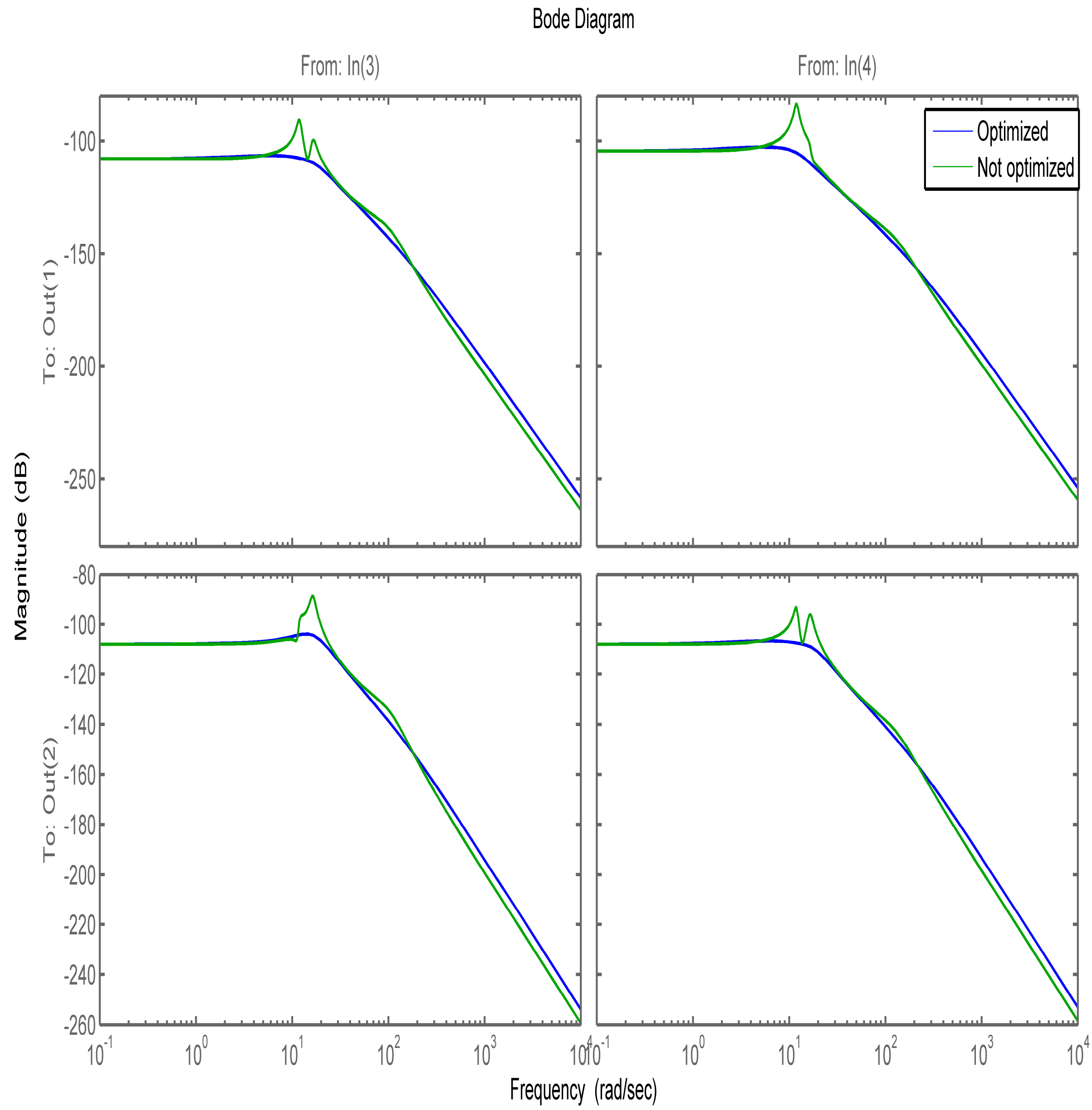

Figure 3: Amplitude characteristic of the solution of minimize criterion

The parameters of the model are

m=1000 kg, m_3=50 kg, m_4=30 kg,

I_T=900 kgm^2, l_1=1,5 m, l_2=1 m,

k_1=4.10^5 Nm^{-1}, k_2=10 Nm^{-1}, b_1=3200 Nsm^{-1},

b_1=800 Nsm^{-1}, s=0,01m, Ω=56,5 s^{-1}.

On the Fig. 4 are shown the displacements of four degrees of freedom x, φ, x_3, x_4 after using the minimize criterion (13). The maximum amplitude of the displacement x of the car body is 0.625 mm. The maximum amplitude of the displacement φ of the car body is 1,875.10^4 rad. The maximum amplitude of the displacement x_3 of the axle 3 is 3,5 mm. The maximum amplitude of the displacement x_4 of the axle 4 is 3,5 mm.

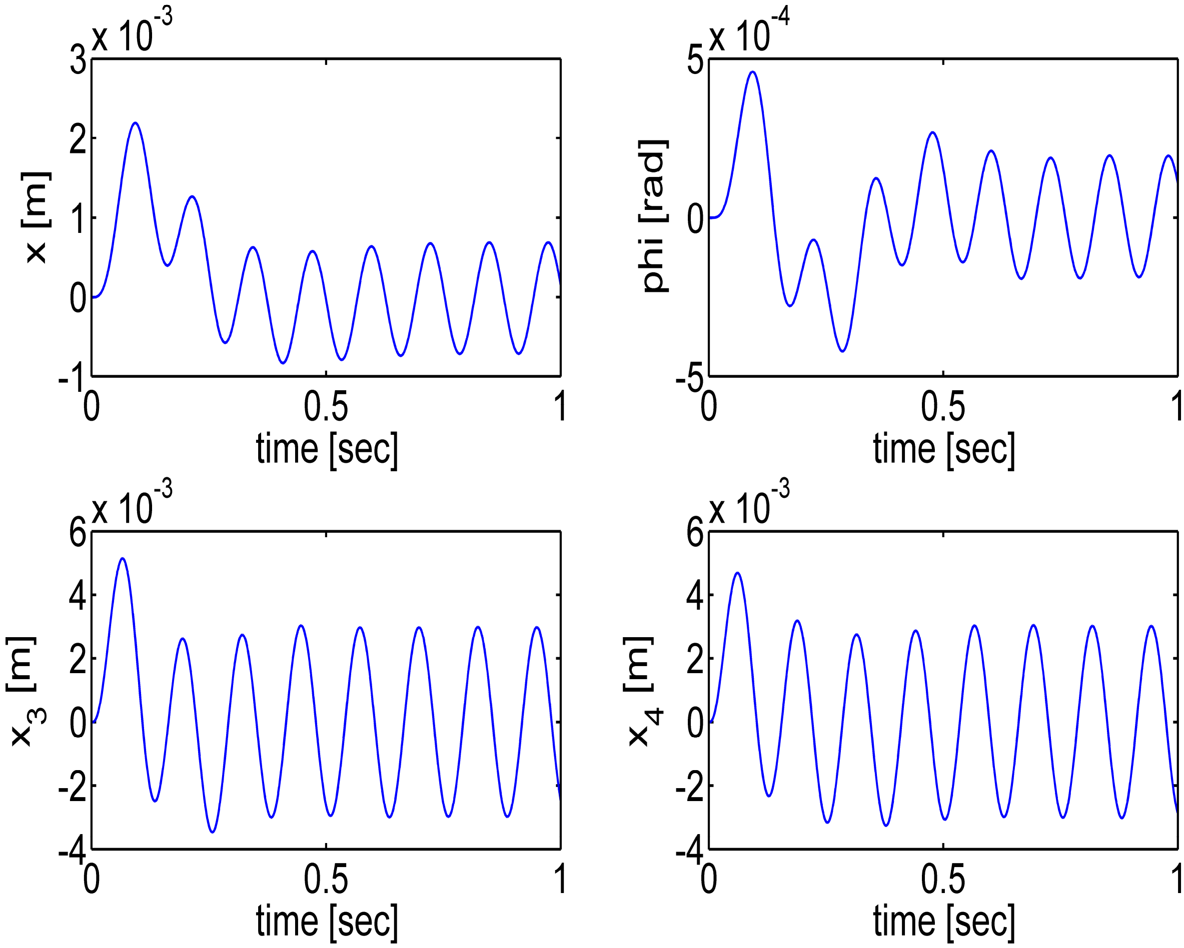

Figure 4: Displacements of the car body x, φ and displacements of the car axles x_3 and x_4

References

[1] http://www.mech.uwa.edu.au/bjs/Vibration/TwoDOF/Absorbers/Optimization/default.html

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