MODELLING AND EXPERIMENTAL ANALYSIS OF MOTORCYCLE DYNAMICS USING MATLAB

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Abstract

The first part of this paper is devoted to mathematical modelling of motorcycle dynamics. A nonlinear 5DOF model of a motorcycle is developed using Lagrange equations. This approach enables to simulate large displacement analyses as well as the contact between tires and road. The set of nonlinear differential equations is solved using constant average acceleration method in MATLAB. The results of multibody dynamics simulation are verified using VI-Motorcycle plugin for Adams/Car and they are used as boundary conditions for FEM analyses.

The second part of this paper is focused on experimental analysis of motorcycle dynamics. A measuring system containing three accelerometers was developed and used for rider vibration exposure analysis. Data from 12.6-kilometer testing track were processed using MATLAB and evaluated according to ČSN ISO 2631.

1 Nonlinear mathematical model of a motorcycle

First step of the solution process is to develop a nonlinear mathematical model of a motorcycle. A planar 5-degree-of-freedom model was proposed with following generalized coordinates: x2 – frame horizontal translation; y2 – frame translation; φ2 – frame pitch angle; y3 – front assembly vertical translation; y4 – rear assembly vertical translation. A scheme of the model containing generalized coordinates as well as other parameters is shown in Fig. 1.

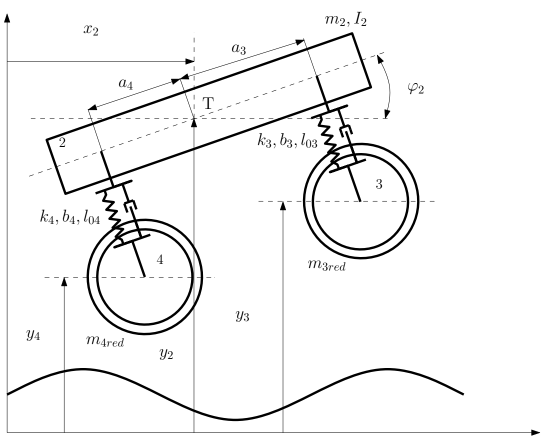

Figure 1: Scheme of the model

The generalized coordinates are included in vector q as shown in equation 1.

$$\mathbf{q} = [x_2, y_2, \varphi_2, y_3, y_4]^T \tag{1}$$

Lagrange equations (eq. 2) were used to develop equations of motion of the system.

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_i}\right) - \frac{\partial E_k}{\partial q_i} + \frac{\partial E_p}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = Q_i \tag{2}$$

Where E_k, E_p and R represent kinetic energy function, potential energy function and Rayleigh dissipation function of the system.

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & m_{3red} & 0 \\ 0 & 0 & 0 & 0 & m_{4red} \end{bmatrix} \tag{3}$$

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & k_3 + k_4 & \frac{(k_3 a_3 - k_4 a_4)\sin\varphi_2}{\varphi_2} & -k_3 & -k_4 \\ 0 & (k_3 a_3 - k_4 a_4)\cos\varphi_2 & \frac{(k_3 a_3^2 - k_4 a_4^2)\sin 2\varphi_2}{2\varphi_2} & -k_3 a_3 \cos\varphi & -k_4 a_4 \cos\varphi \\ 0 & -k_3 & \frac{-k_3 a_3 \sin\varphi_2}{\varphi_2} & k_3 & 0 \\ 0 & -k_4 & \frac{k_4 a_4 \sin\varphi_2}{\varphi_2} & 0 & k_4 \end{bmatrix} \tag{4}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & b_3 + b_4 & (b_3 a_3 - b_4 a_4)\cos\varphi_2 & -b_3 & -b_4 \\ 0 & (b_3 a_3 - b_4 a_4)\cos\varphi_2 & (b_3 a_3^2 - b_4 a_4^2)\cos^2\varphi_2 & -b_3 a_3 \cos\varphi_2 & b_4 a_4 \cos\varphi_2 \\ 0 & -b_3 & -b_3 a_3 \cos\varphi_2 & b_3 & 0 \\ 0 & -b_4 & b_4 a_4 \sin\varphi_2 & 0 & b_4 \end{bmatrix} \tag{5}$$

The stiffness matrix and the damping matrix are both not constant and depend on the generalized coordinate φ2 which means the system is nonlinear and able to provide more accurate results as displacements increase. However, this also places requirements on the integration method of the motion equations itself.

1.1 Tire contact model

Interaction between tires and road is a significant source of forces that act on the frame. In this case longitudinal driving and braking forces are neglected and the key task is to obtain normal contact forces. As stated previously the whole model is planar therefore lateral cornering forces are not taken into account either. The model of the tire contact is shown in Figure 2.

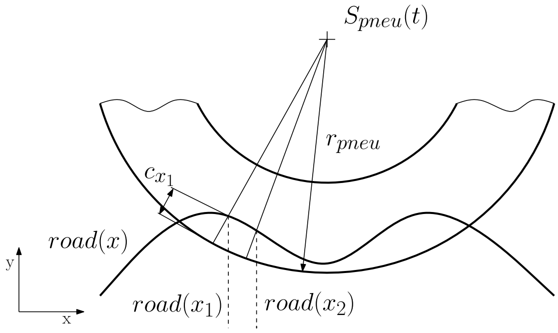

Figure 2: Tire contact model

Let's assume that the location of tire center in known at any time. In case that any point of road surface is closer to the tire center than tire radius a contact pressure exerting on the tire can be calculated as

$$p_k = k_p c \tag{6}$$

Where k_p describes stiffness characteristics of the tire and c is the difference between the tire radius and the distance. Differential normal force can be obtained according to eq. 6.

$$dN = k_p c ds \tag{7}$$

Differential ds relates to tire arc. Differential normal force is always perpendicular to the road surface. For our problem it is crucial to determine its components in horizontal and vertical direction. This can be accomplished by using derivative of road function as described in the following equations.

$$dN_x = k_p c \sin\left(\arctan \frac{droad}{dx} \right) dx \tag{8}$$

$$dN_x = k_p c \cos\left(\arctan \frac{droad}{dx} \right) dx \tag{9}$$

In order to determine the components of the actual normal force, integration is conducted as the very last step.

$$N_x = \int_{x(S_{pneu})-r_{pneu}}^{x(S_{pneu})+r_{pneu}} dN_x \tag{10}$$

$$N_y = \int_{y(S_{pneu})-r_{pneu}}^{y(S_{pneu})+r_{pneu}} dN_y \tag{11}$$

The tire contact model proposed above is based on elastic contact model. Tire hysteresis effects can be taken into consideration as well assuming damping forces have similar character to normal contact forces as stated in the following equation.

$$N_b = \dot{c} N \frac{b_k}{k_k} \tag{12}$$

The variable $\dot{c}$ from previous equation relates to tire deformation rate.

2 Numerical integration of the equations of motion

Once we acquired the forces acting on tires we can proceed to the solution process of the motion equations with MATLAB. Set of 5 nonlinear ordinary differential equations is summed up in the Equation 12.

$$\mathbf{M}\ddot{\mathbf{q}}_{(t)} + \mathbf{B}\dot{\mathbf{q}}_{(t)} + \mathbf{K}\mathbf{q}_{(t)} = \mathbf{f}_n(\dot{\mathbf{q}}, \mathbf{q}, t) \tag{13}$$

Constant average acceleration integration method was used due to its relative simplicity and numerical stability. This method is predictor-corrector based as the system in question is nonlinear. A road function was designed using modified sin functions to represent an obstacle which is supposed to cause a loss of contact between the tires and the road. In this way a jump of a motorcycle can be simulated.

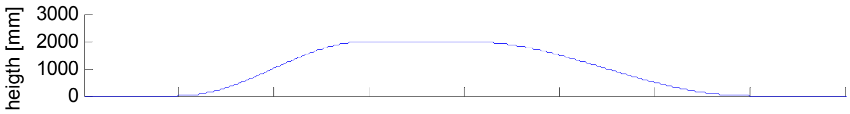

Figure 3: Road profile

Constant average acceleration method is based on an assumption that the acceleration is constant between two points in time (Equation 13).

$$\ddot{\mathbf{q}} = \frac{1}{2}\left(\ddot{\mathbf{q}}_t + \ddot{\mathbf{q}}_{t+\Delta t}\right) \tag{14}$$

By integration of the previous equation we obtain formulas for velocities and displacements.

$$\dot{\mathbf{q}}_{t+\Delta t} = \dot{\mathbf{q}}_t + \frac{\Delta t}{2}\left(\ddot{\mathbf{q}}_t + \ddot{\mathbf{q}}_{t+\Delta t}\right) \tag{15}$$

$$\mathbf{q}_{t+\Delta t} = \mathbf{q}_t + \dot{\mathbf{q}}_t \Delta t + \frac{\Delta t^2}{4}\left(\ddot{\mathbf{q}}_t + \ddot{\mathbf{q}}_{t+\Delta t}\right) \tag{16}$$

Substituting Equations 14 and 15 into Equation 12 leads to:

$$\mathbf{Z}\ddot{\mathbf{q}}_{t+\Delta t} = \tilde{\mathbf{f}}_{t+\Delta t} \tag{17}$$

Therefore the acceleration in the next time step is calculated as:

$$\ddot{\mathbf{q}}_{t+\Delta t} = \mathbf{Z}^{-1}\tilde{\mathbf{f}}_{t+\Delta t} \tag{18}$$

Acceleration results are shown in the figures below. In Figure 4 there are acceleration results of the frame. The initial steep increase relates to the beginning of the obstacle. As the acceleration drops to approximately -10 ms^{-2} it means that the tires lost contact with surface. The peak values occur after landing reaching 36 ms^{-2}. Acceleration results of front and rear assembly in Figure 5 can be explained in a similar way.

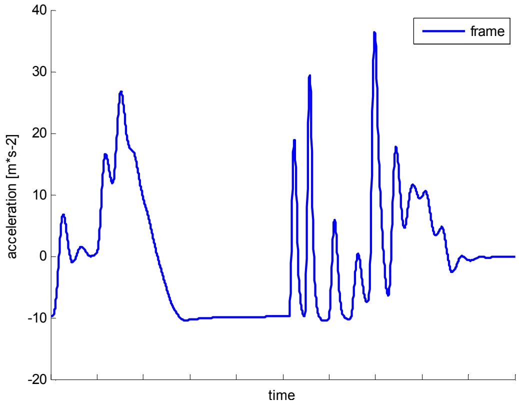

Figure 4: Frame acceleration results

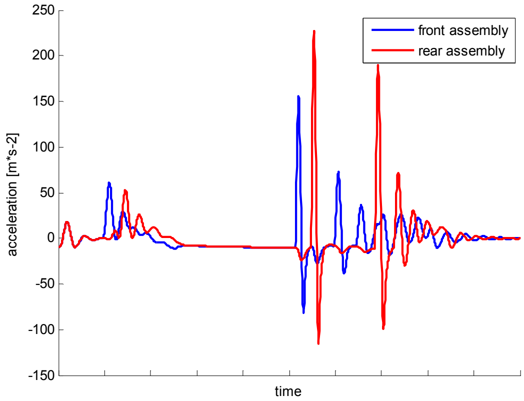

Figure 5: Front and rear acceleration results

The mathematical model that was developed provides reasonable results; however, further verification is necessary. In order to do so, ADAMS/VI-Motorcycle module was used to compare results. In Figure 6 there is a motorcycle model in VI-Motorcycle interface running over an obstacle defined in the same as in the MATLAB model. In Figure 7 there are acceleration results of front assembly in ADAMS and MATLAB. In this simulation tire damping properties were neglected in MATLAB which explains the key difference between both curves.

Figure 6: VI-Motorcycle model

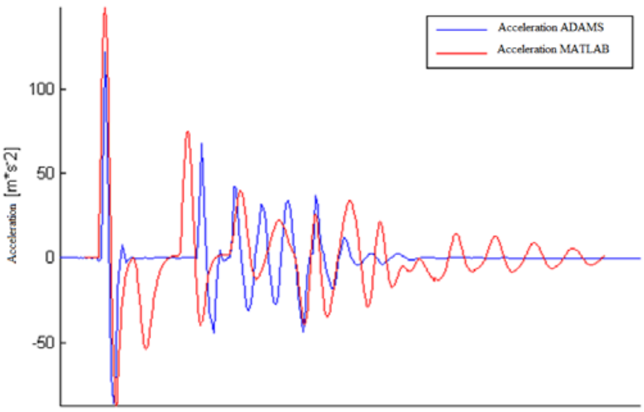

Figure 7: Front assembly acceleration result comparison

3 Experimental analysis

A measurement system was designed in order to fulfill two purposes listed below by using inexpensive parts used in automotive applications.

1) Measuring dynamics of a motorcycle in motion in order to verify computational model
2) Evaluation of motorcycle vibrations effects on the rider by application of ČSN ISO2631 and ČSN EN ISO 5349 standards

For data logging DL1 MK3datalogger was chosen. Advantages of this datalogger are the possibility of connecting up to 12 external sensors and the possibility of logging data from vehicle control unit.

Figure 8: Datalogger DL1 MK3 [2]

In order to obtain relationship between output voltage of accelerometers and values of acceleration a simple experiment was done. During this experiment each axis was exposed to positive and negative gravitational acceleration as well as to zero acceleration. By statistic evaluation of these three points a linear relation between output voltage and amount of acceleration was obtained.

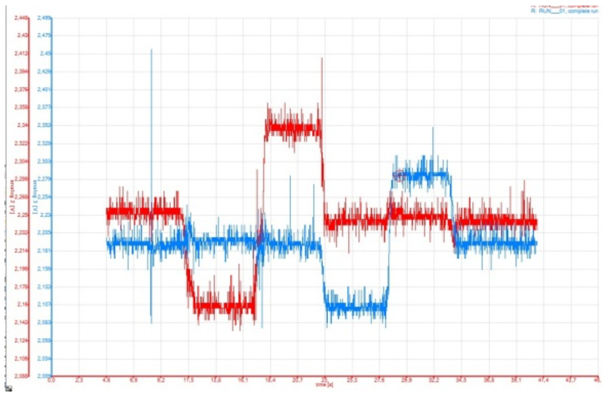

Figure 9: Output signal of two axis accelerometer

3.1 Program N. 1 – Processing of the measured digital signals

To simplify the processing of measured signals a GUI was created. Main purpose of this program is to convert measured signals from voltage to units of acceleration by using previously computed linear relation. The GUI also has functions aimed mainly at verifying function of sensors and evaluation of measured data.

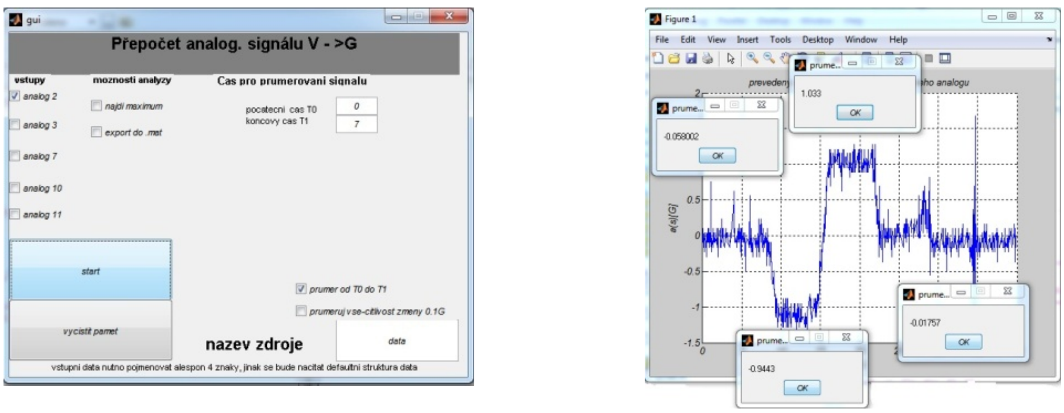

Figure 10: GUI and evaluation of measured data

3.2 Program N. 2 – Evaluation of data by the methodic of ČSN ISO 2631 standard

This program is processing output data of program N. 1 by using ČSN ISO 2631 standard, aimed at effects on health and comfort. The outputs of this program are a frequency spectrum of every axis of accelerometers, summed up amount of vibrations (for comparing with limit amount a_c), effective values of weighted vibrations $a_{hw\ j}$ and a controlling factor. Verification of this program was made by the following generated input signal:

$$x = 0{,}7\sin(2\pi 50t) + \sin(2\pi 100t) + \sin(2\pi 190t) \tag{19}$$

$$y = x + 2\,\mathrm{rand}\,n(\mathrm{size}(t)) \tag{20}$$

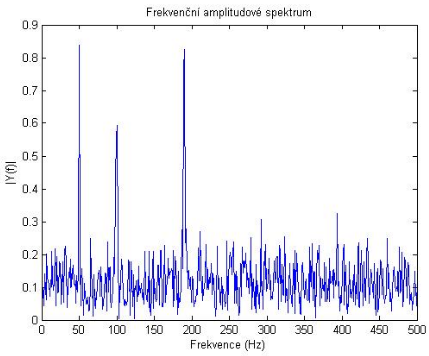

Figure 11: Output of the FFT algorithm obtained by using generated input

3.3 Program N. 3 – Evaluation of data by the methodic of ČSN EN ISO 5349 standard

This program is processing output data of program N. 1 by using ČSN EN ISO 5349 standard. Outputs of this program are effective values of weighted vibrations and total daily exposition to vibrations A(8). This evaluation process can be used only as a reference to compare different motorcycles. Precise evaluation is not relevant because the values recommended by this standard are not meant for motorcycles.

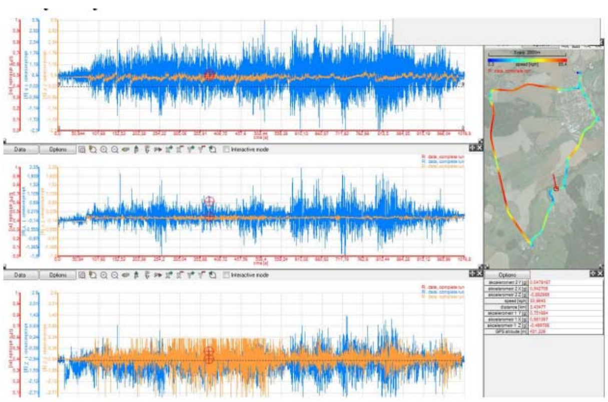

Figure 12: Raw measured data in sw racetechnology Analysis 8.5

4 Conclusion

The mathematical model of a motorcycle that was developed is in good agreement with commercial software VI-Motorcycle. Its outputs might be used for consequential FEM simulations. Further development of this model is planned in order to conduct three dimensional analyses.

As far as experimental analysis is concerned, correct function of developed programs was tested on data obtained by measuring dynamics of the real motorcycle riding on public roads. Unfortunately, usage of this system is nowadays limited by insufficient sampling frequency of datalogger and low bandwidth of used accelerometers. Both issues are currently being solved.

References

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