

ANALYSIS OF CIRCUITS EXCITED BY PERIODIC NONSINUSOIDAL SIGNALS IN MATLAB

I. Tomčíková

Technical university in Košice, Slovakia

Abstract

Analysis of circuits excited by periodic nonsinusoidal signals can be done in the time domain or in the complex frequency domain, but in the both cases, it is time-consuming activity. In the time domain, finding the complete response of such circuits means to solve a dynamic circuits that have not only resistors but also dynamic elements, and the circuit equations take the form of integrodifferential equations with the right-hand side having the form of the periodic function. In the complex frequency domain, the analysis of these circuits is done after transforming the circuits with their initial conditions directly into the complex frequency domain using the Laplace transform. Then it can be dealt with them as if they consisted of sources and resistors only, because the passive elements have impedances. Combining the last procedure together with sparse tableau analysis and the symbolic computation of MATLAB, there is no difficulty in analysis of dynamic circuits, because formulation of the circuit equations, solving the obtained equations, and taking the inverse Laplace transform are executed in MATLAB environment.

1 Principles of Analysis of Electric Circuits Excited by Periodic Nonsinusoidal Signals

Analysis of electric circuits excited by periodic nonsinusoidal signals in the time domain implies the problem of solving a set of integrodifferential circuit equations with the right-hand side having the form of the periodic function in order to find all branch currents and voltages. After transforming the circuit with its initial conditions directly into the complex frequency domain using the Laplace transform, the analysis of such circuits implies the problem of solving a set of algebraic equations because the passive elements have impedances, which can be regarded as generalized resistances. To obtain a complete description of a general circuit, the set of circuit equations is necessary. This set consists of the equations depending on the topology of the circuit and the equations depending on the type of the circuit elements. The equations, depending on the topology of the circuit, represent how the circuit elements are connected to one another in the circuit. These equations are called the connection equations. The equations, which depend on the type of the circuit element, describe the voltage-current relationship and they are called the element equations or branch equations. If the voltage-current characteristics for two-terminal elements and two-port elements in the circuit are linear, and time-invariant, then the set of the equations describing the circuit, is also linear.

The best way for circuit equations formulation is to use a method that enables systematic and automatic formulation of the circuit equations. For such purposes, two methods are appropriate: modified nodal analysis and sparse tableau analysis [3]. Sparse tableau analysis is most general formulation of the equations describing the circuit because the solution provides the currents through all elements, the voltages across all elements and all nodal voltages simultaneously. This method consists of the connection equations and the element equations.

The connection equations are obtained by applying Kirchhoff's laws to the circuit, which leads to the two sets of linear algebraic equations in terms of the branch currents and the branch voltages. The set of connection equations must be linearly independent.

The first set of the connection equations can be expressed [1] – [3]:

$$\mathbf{A} \mathbf{i} = \mathbf{0}, \quad (1)$$

where \mathbf{i} is a matrix of branch currents, \mathbf{A} being the node versus branch reduced-incidence matrix.

The second set of the connection equations can be expressed in terms of the branch voltages using the fundamental loop versus branch incidence matrix. The better way is to convey this set of the equations in terms of the branch voltages and the node voltages as [3]:

$$\mathbf{u} = \mathbf{A}^T \mathbf{v}, \quad (2)$$

where \mathbf{A}^T is the transposed matrix \mathbf{A} , \mathbf{u} being the branch voltage vector, \mathbf{v} being the node voltage vector.

The element equations are related according to the voltage-current characteristics of the elements. For linear circuits, containing the ideal operational amplifiers too, the element equations can be expressed [3]:

$$\mathbf{K}_u \mathbf{u} + \mathbf{K}_i \mathbf{i} = \mathbf{s}, \quad (3)$$

where $\mathbf{K}_u, \mathbf{K}_i$ are the matrices containing the coefficients that define the linear voltage-current relationships for the circuit elements uniquely, \mathbf{s} being the vector containing the parameters of the independent voltage and current sources.

Assembling the equations (1), (2), and (3), the sparse tableau equations are constituted. It is convenient to rewrite the sparse tableau equations as a single matrix equation [3]:

$$\mathbf{T} \mathbf{x} = \mathbf{w}, \quad (4)$$

where \mathbf{T} is the square tableau matrix, \mathbf{x} being the vector of unknown variables, \mathbf{w} being the vector containing zero vectors of appropriate dimensions and the vector \mathbf{s} .

The Laplace transform for a time-domain function $f(t)$ (must be zero for $t < 0$) of a real variable t is [1], [2]:

$$\hat{f}(p) = \mathcal{L}\{f(t)\} = \int_0^{\infty} f(t) e^{-pt} dt, \quad (5)$$

where p is a complex variable (meaning the complex frequency, $p = \sigma + j\omega$), and $\hat{f}(p)$ is a complex frequency function.

The inverse Laplace transform is defined by the integral [1], [2]:

$$f(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} \hat{f}(p) e^{pt} dp, \quad (5)$$

where $j = \sqrt{-1}$ is an imaginary unit.

Applying the Laplace transform to the equations (1), (2), (3), and (4) the following equations are obtained:

$$\mathbf{A} \hat{\mathbf{i}} = \mathbf{0}, \quad (6)$$

where $\hat{\mathbf{i}}$ is a vector of the complex frequency-domain representation of branch currents of the circuit,

$$\hat{\mathbf{u}} = \mathbf{A}^T \hat{\mathbf{v}}, \quad (7)$$

where $\hat{\mathbf{u}}$ is a vector of the complex frequency-domain representation of branch voltages of the circuit, $\hat{\mathbf{v}}$ being a vector of the complex frequency domain of node voltages of the circuit,

$$\hat{\mathbf{K}}_u \hat{\mathbf{u}} + \hat{\mathbf{K}}_i \hat{\mathbf{i}} = \hat{\mathbf{s}}, \quad (8)$$

where $\hat{\mathbf{K}}_u, \hat{\mathbf{K}}_i$ are the matrices containing the coefficients that define the complex frequency domain linear voltage-current relationships for the circuit elements uniquely, $\hat{\mathbf{s}}$ being the vector containing the complex frequency-domain parameters of the independent voltage and current sources and the initial conditions.

The sparse tableau equations expressed as the single matrix equation in the complex frequency domain using the Laplace transform is

$$\hat{\mathbf{T}} \hat{\mathbf{x}} = \hat{\mathbf{w}}, \quad (9)$$

where $\hat{\mathbf{T}}$ is the square tableau matrix expressed in the complex frequency domain, $\hat{\mathbf{x}}$ being the vector of unknown variables in the complex frequency domain, $\hat{\mathbf{w}}$ being the vector containing zero vectors of appropriate dimensions and the vector \mathbf{s} in the complex frequency domain.

Procedure of finding the complete response of the circuit to periodic input signal involves:

- to find the complex frequency-domain representation (using the Laplace transform) of the input periodic signal having a fundamental period T_{period} and working on the presumption that its time domain representation is zero for all $t < 0$,
- to find the complex frequency-domain representation of the circuit elements using the Laplace transform,
- to determine the transient part of the complete response for $t > 0$ at first in the complex frequency domain, and next to take the inverse Laplace transform of it in order to obtain the time-domain representation the transient part of the complete response for $t > 0$,
- to determine the complete response over the interval $(0, T_{\text{period}})$ at first in the complex frequency domain and next to take the inverse Laplace transform of it in order to obtain the time-domain representation the complete response over the interval $(0, T_{\text{period}})$,
- to determine the steady-state response over the interval $(0, T_{\text{period}})$ at first in the complex frequency domain, and next to take the inverse Laplace transform of it in order to obtain the time-domain representation the steady-state response over the interval $(0, T_{\text{period}})$,
- to find the steady-state response for $t > 0$ in the time domain,
- finally, to find the time-domain representation of the complete response of the circuit for $t > 0$.

All the above steps, which must be done to find the complete response of dynamic circuit, can be easily executed by running the program composed in the MATLAB environment.

2 Results

The proposed procedure for analysis of circuits excited by periodic nonsinusoidal sources is applied to the circuit shown in Figure 1 – left. The voltage, $u(t)$, of the voltage source is the periodic square wave shown in Figure 1 – right. The circuit is at steady state before the input voltage $u(t)$ is applied at time $t = 0$. The aim is to find all the element currents and voltages for $t > 0$ if all the initial conditions are equal zero.

The corresponding graphical representations of the transient and steady-state responses are given for the following values of the circuit parameters: $R = 100 \Omega$, $L = 0.012 \text{ H}$, $C = 2 \mu\text{F}$, $T = 0.008 \text{ s}$, $U = 25 \text{ V}$, and they are shown in figures Figure 2 to Figure 4.



Figure 1: The second-order circuit (left) and the input voltage signal expressed in graphical form (right)

In Figure 2 (left), the current, $i_L(t)$, flowing through the inductor is depicted. The plot shows that steady-state values of the inductor current are 0 A and about 0.25 A. It is evident that the transient response of the current obtains very small values. Thus, the steady-state response is equal to the complete response of the inductor current because of that. In Figure 2 (right), the voltage, $u_L(t)$, across the inductor is drawn. The plot shows that steady-state value of the inductor voltage is 0 A (at steady state the inductor acts as a short circuit).

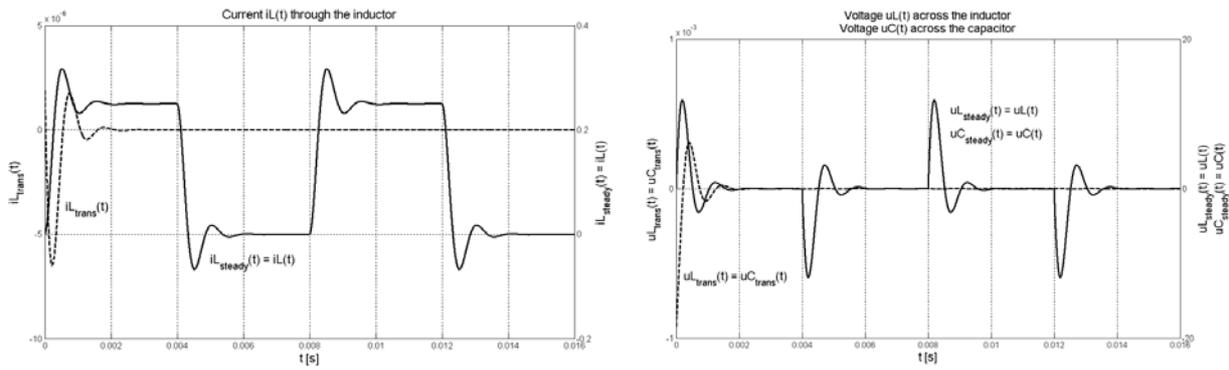


Figure 2: MATLAB plots of the inductor current, $i_L(t)$, (left) and the inductor voltage, $u_L(t)$, (right) for the circuit shown in Figure 1

In Figure 3 (left), the current, $i_C(t)$, flowing through the capacitor is depicted. The plot shows that steady-state value of the capacitor current is 0 A (at steady state the capacitor acts as an open circuit). In Figure 3 (right), the capacitor voltage, $u_C(t)$, is shown. This graph is the same as for the inductor voltage because the inductor and capacitor are connected in parallel.

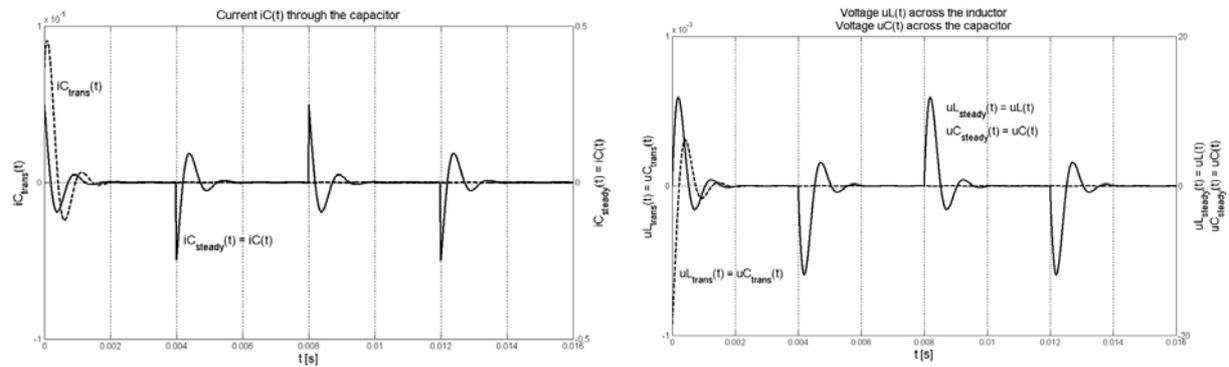


Figure 3: MATLAB plots of the capacitor current, $i_C(t)$, (left) and the capacitor voltage, $u_C(t)$, (right) for the circuit shown in Figure 1

In Figure 4 (left), the current, $i_R(t)$, flowing through the resistor is drawn. The plot shows that steady-state values of the resistor current is 0 A and about 0.25 A (at steady state the inductor acts as a short circuit, the capacitor acts as an open circuit, thus the same current flows through the resistor and inductor). In Figure 4 (right), the voltage, $u_R(t)$, across the resistor is depicted. Its plot is identical to the graph of the current $i_R(t)$, flowing through the resistor (due to the voltage-current relationship for the resistor).

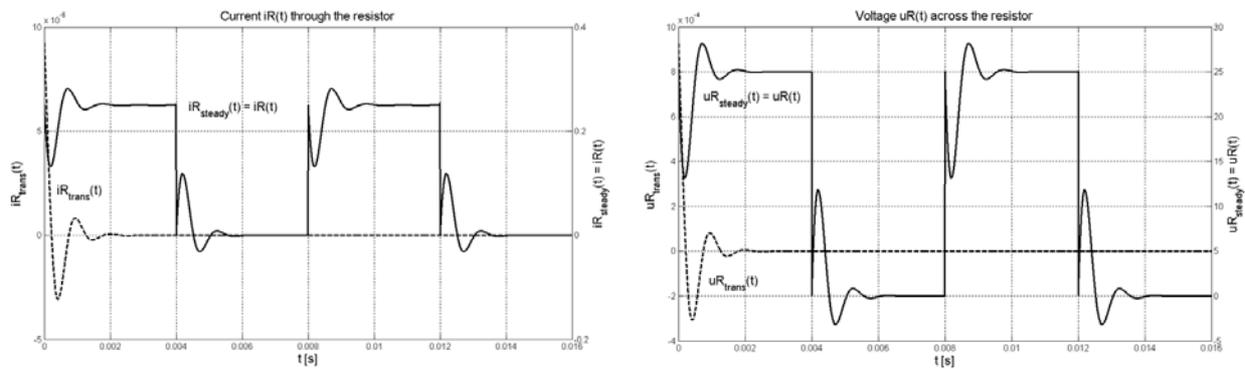


Figure 4: MATLAB plots of the resistor current, $i_R(t)$, (left) and the resistor voltage, $u_R(t)$, (right) for the circuit shown in Figure 1

It is evident that all the responses of the given circuit given are oscillatory-type responses with decreasing magnitude and the oscillation frequency that depends on the damped resonant frequency.

Obtained results are in accordance with the fact that the analyzed circuit is the underdamped second-order circuit. The time responses of the underdamped circuit take form of oscillatory-type ones. The values of the transient responses are from the range 10^{-3} to 10^{-6} and the complete and steady-state responses from the range 10^{-1} to 10^{+2} . Due to this, all the plots shown in figures Figure 2 to Figure 4 are created with y-axes on both left and right side of the graphs. The two y-axes enable to display both sets of data on one graph even though relative values of the data belong to quite different value ranges.

The program composed in MATLAB environment has many advantages over the calculus made by hand. It saves a lot of time, effort and provides the very simple visualization of obtained results. The elapsed time that represents the processing and execution of the instructions in the composed MATLAB program for given representative example of the circuit analysis takes 156.422 seconds for step 10^{-6} of MATLAB plots that is very short time.

References

- [1] R.C.Dorf, J.A.Svoboda: *Introduction to Electric Circuits*. John Wiley & Sons, Inc., 2007.
- [2] D. Mayer: *Introduction in Theory of Electric Circuits*. SNTL– Prague, 1984 (in Czech).
- [3] F. N. Najm: *Circuit Simulation* – New Jersey: JohnWiley&Sons, Inc., 2010.
- [4] I. Mayer: *Theoretical Electrical Engineering II* (Part 2) – Bratislava: ALFA, 1977 (in Slovak).
- [5] D. Špaldonová, I. Tomčíková: *Electrotechnics. Lectures*. Košice, 2009 (in Slovak).
- [6] MATLAB - User's Guide, MathWorks, 2009.

Iveta Tomčíková

Technická univerzita v Košiciach, Fakulta elektrotechniky a informatiky, Katedra teoretickej a priemyselnej elektrotechniky, Park Komenského 3, 042 00 Košice
e-mail: iveta.tomcikova@tuke.sk